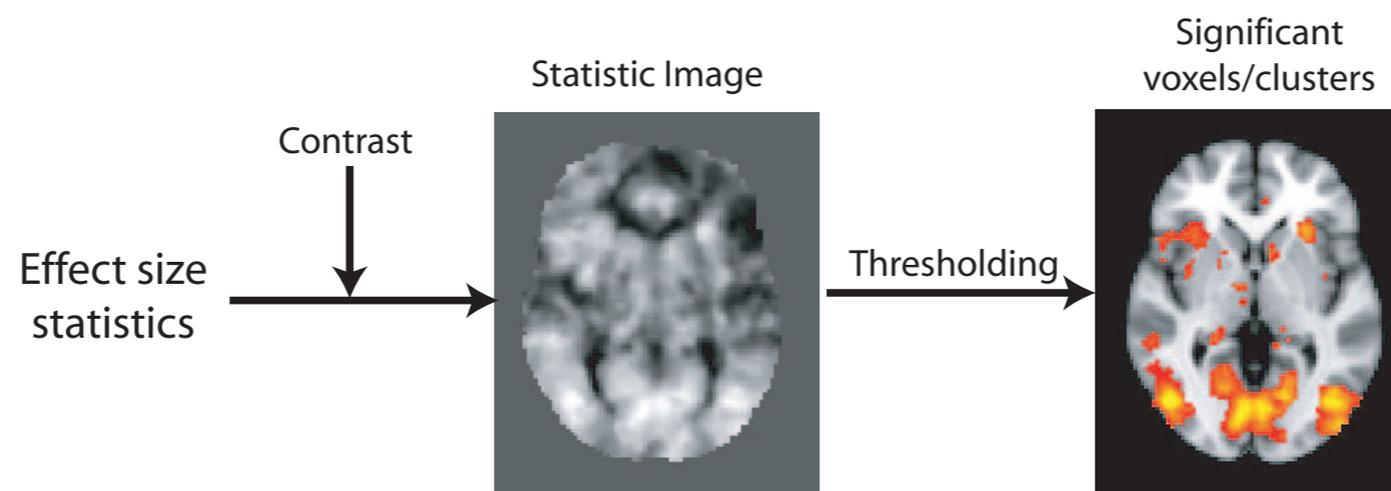
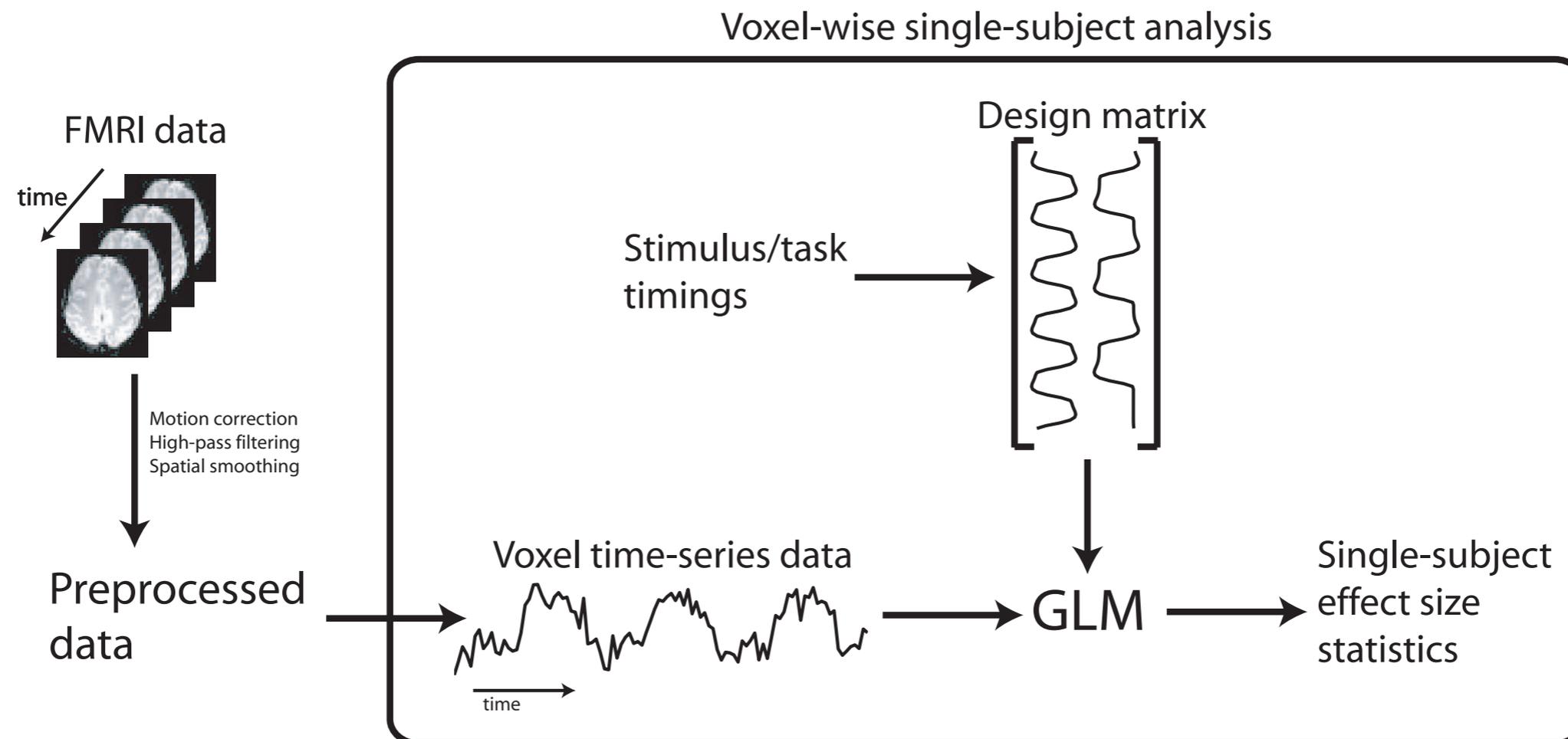




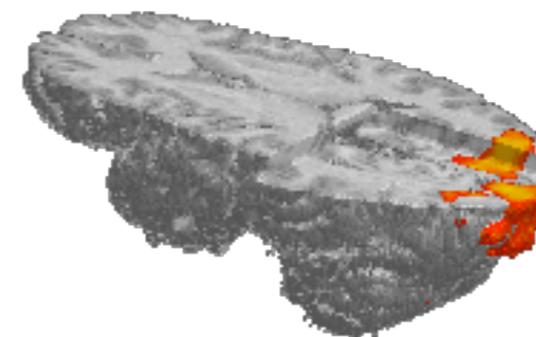
Single-Session Analysis





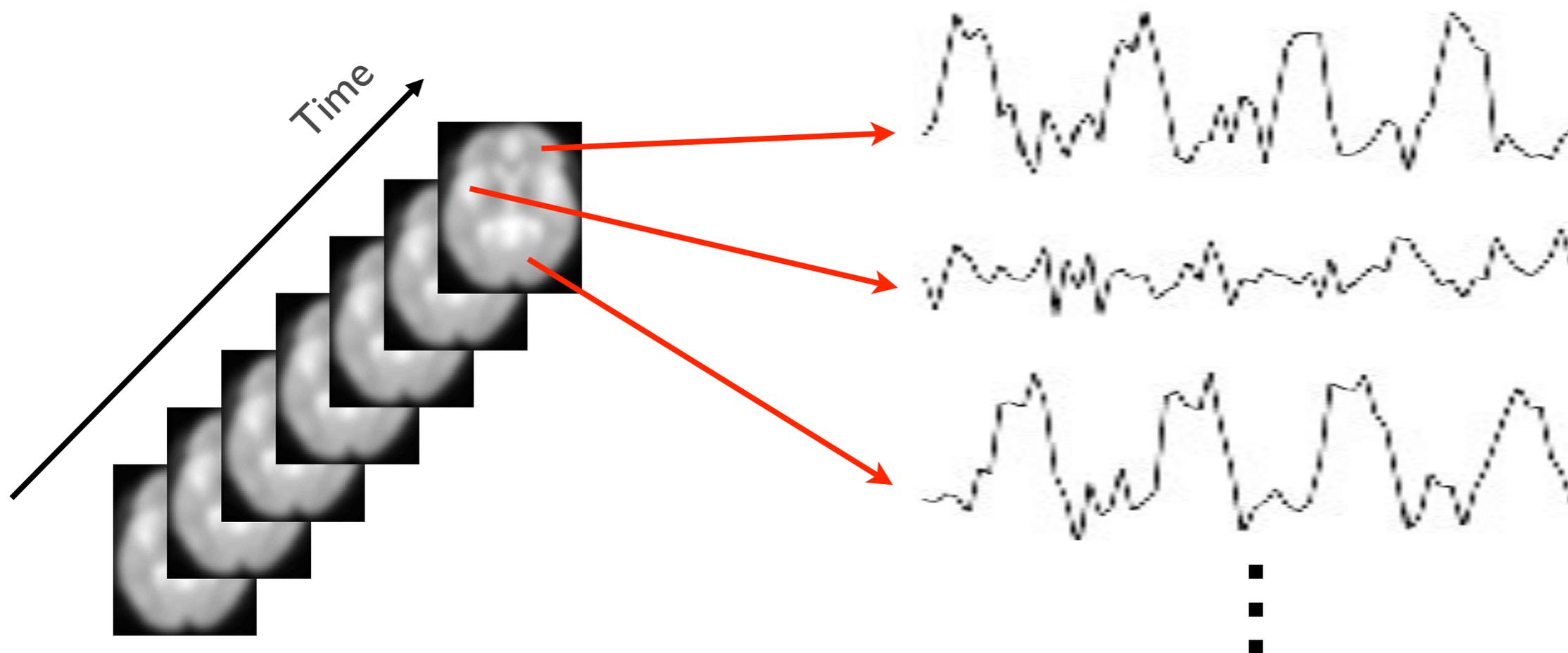
FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





Two different views of the data



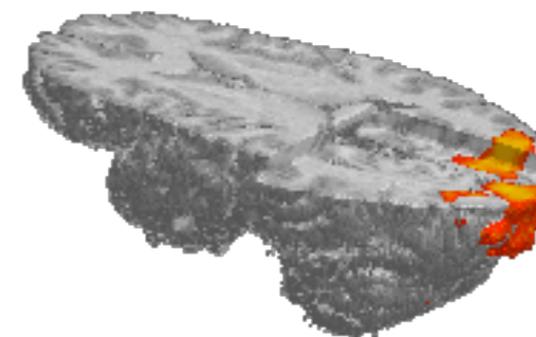
A “smallish”
number of
volumes

A **large**
number of
time series



FMRI Modelling and Statistics

- **An example experiment**
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction

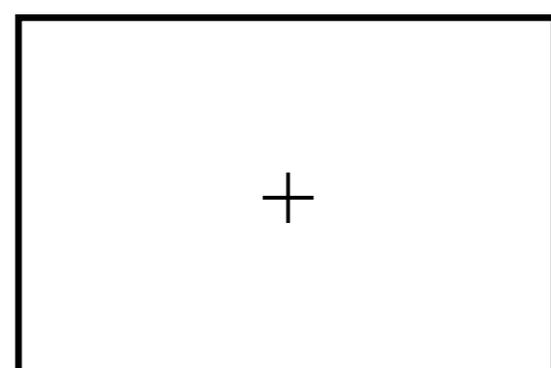




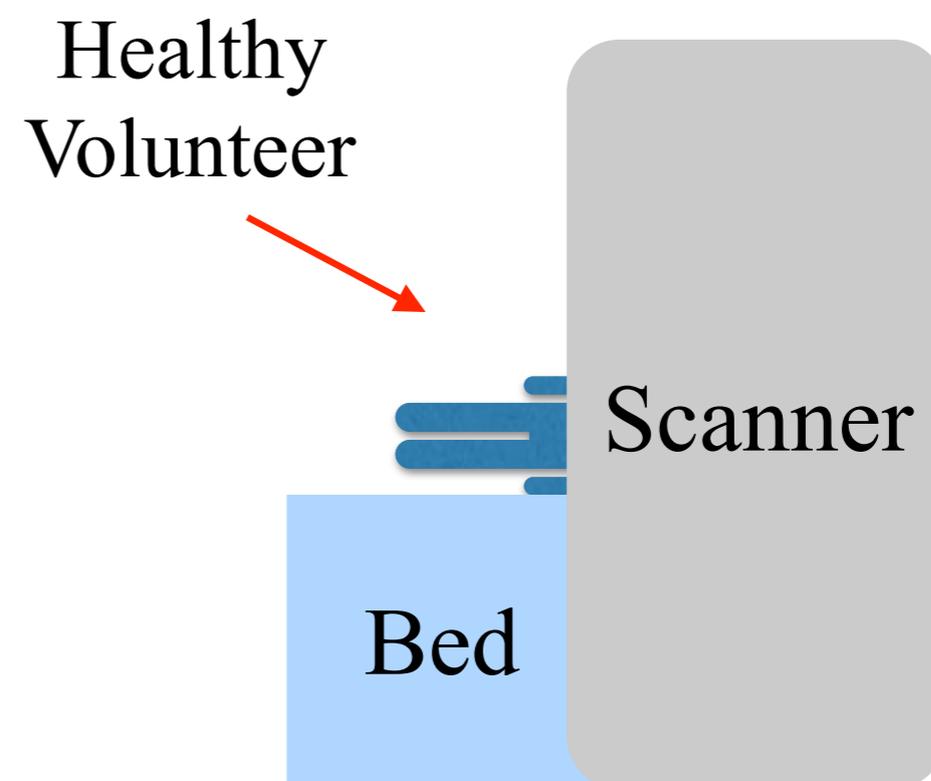
An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation



Screen



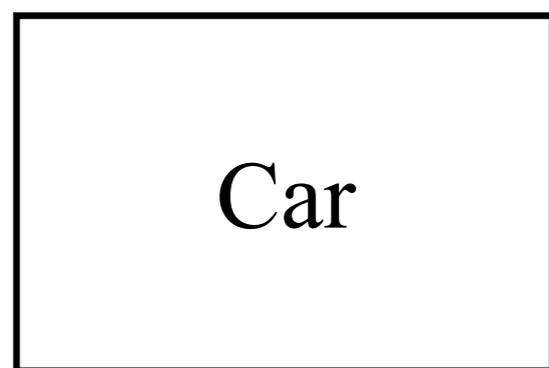


An example experiment

An fMRI adaptation of a classical PET experiment

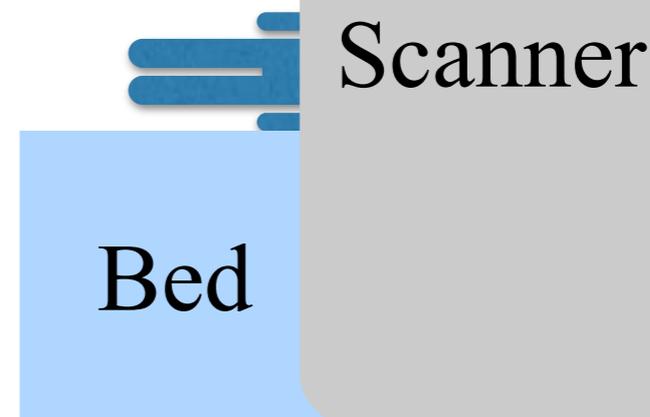
- Three types of events
- 1st type: Word Generation

Noun is presented



Screen

Healthy
Volunteer



Verb is generated





An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation

Noun is presented



Screen

Healthy
Volunteer

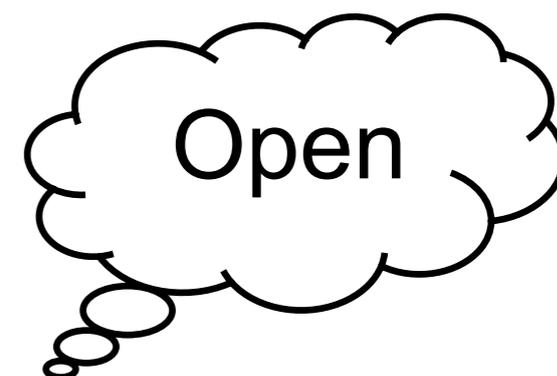


Scanner



Bed

Verb is generated



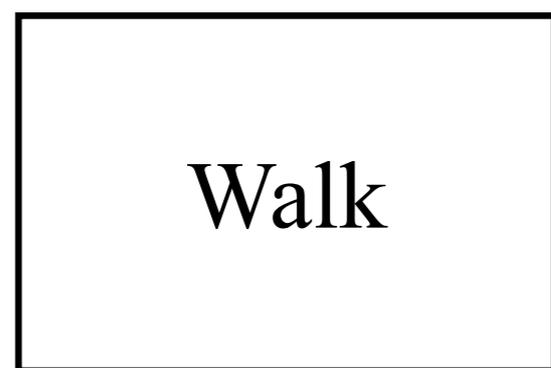


An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing

Verb is presented



Screen

Healthy
Volunteer



Bed



Scanner

Verb is repeated





An example experiment

An fMRI adaptation of a classical PET experiment

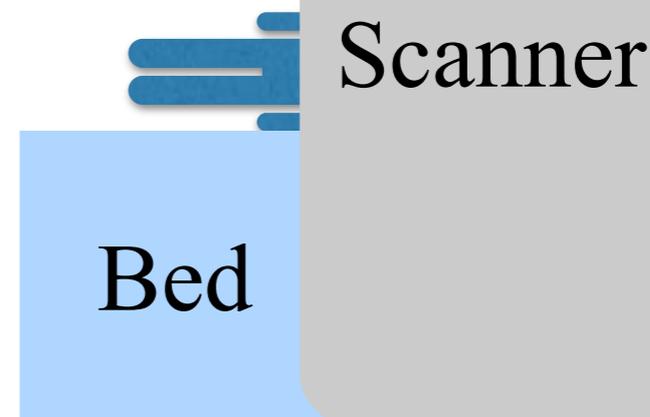
- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing

Verb is presented

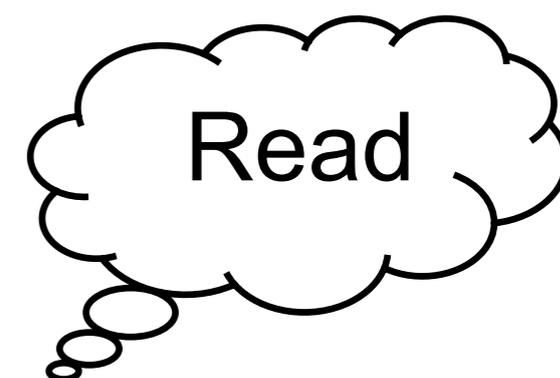


Screen

Healthy
Volunteer



Verb is repeated



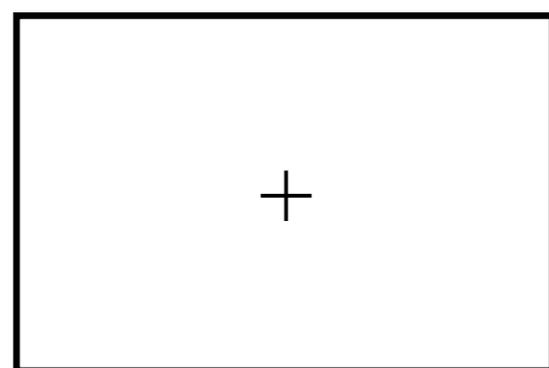


An example experiment

An fMRI adaptation of a classical PET experiment

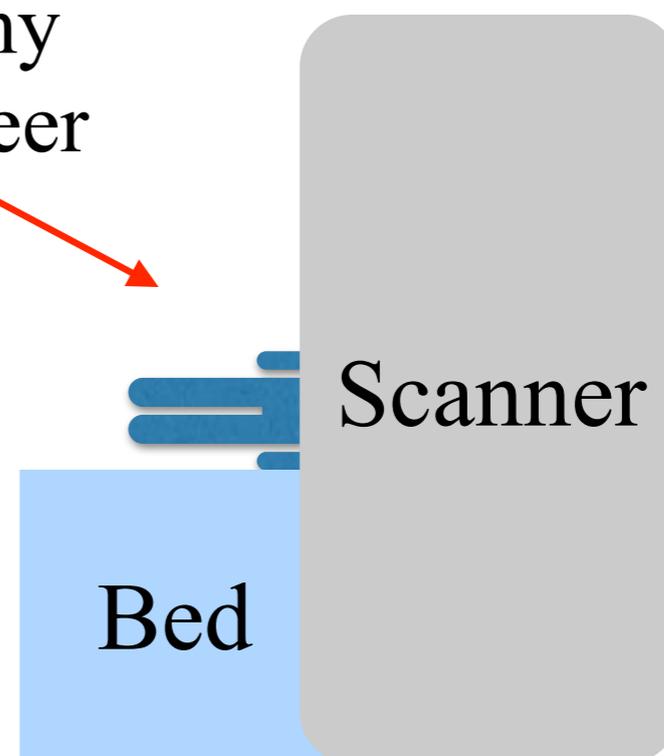
- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event

Crosshair is shown



Screen

Healthy
Volunteer



Scanner

Bed

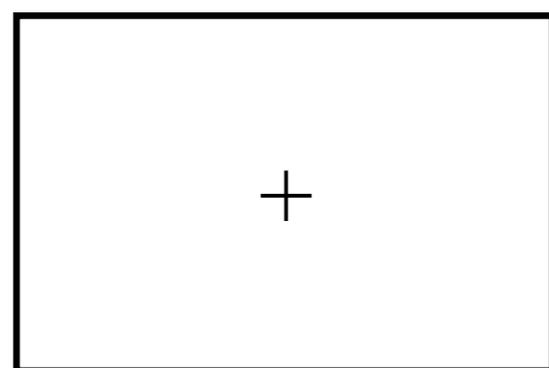


An example experiment

An fMRI adaptation of a classical PET experiment

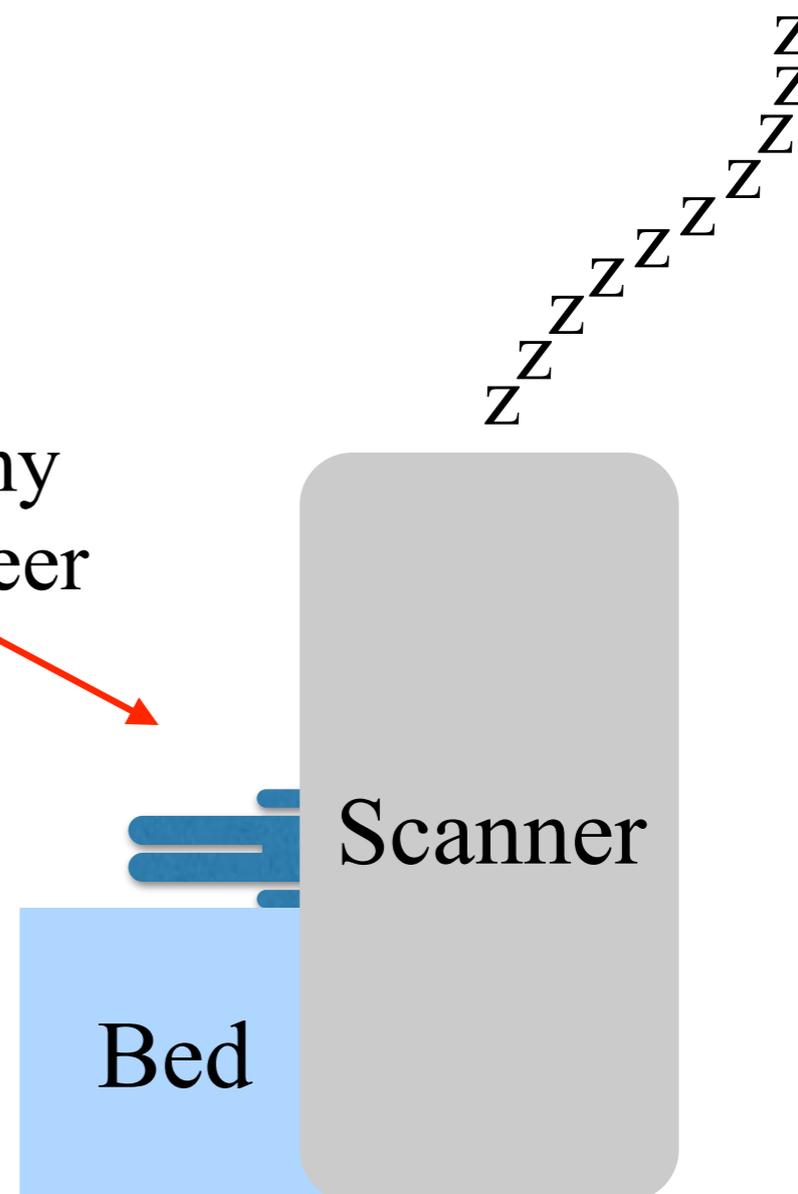
- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event

Crosshair is shown



Screen

Healthy
Volunteer



Scanner

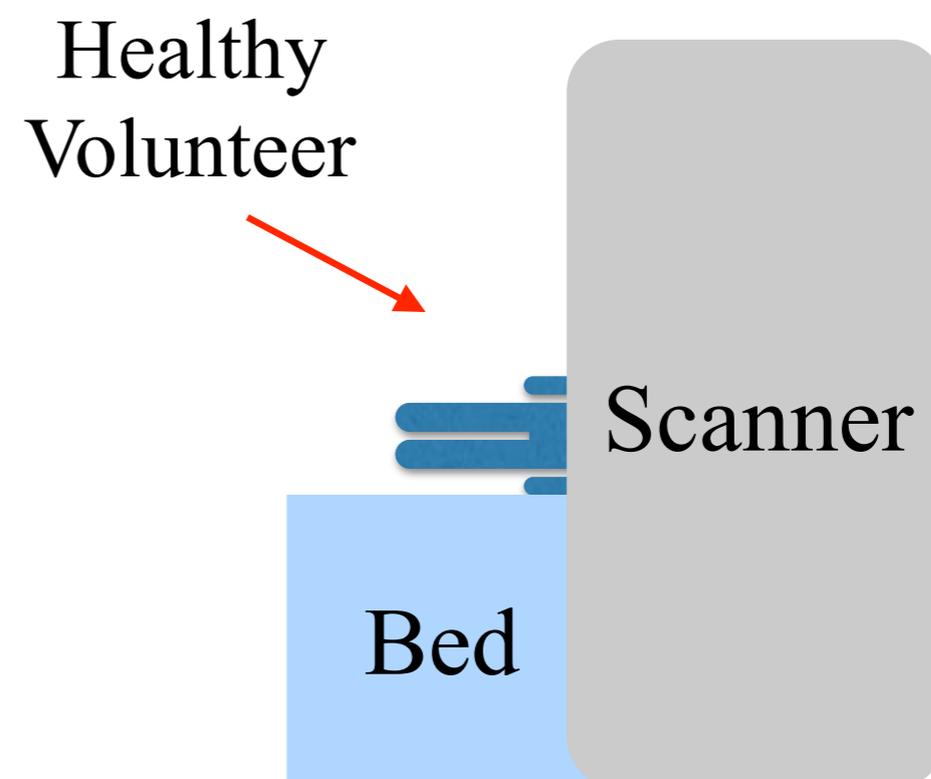
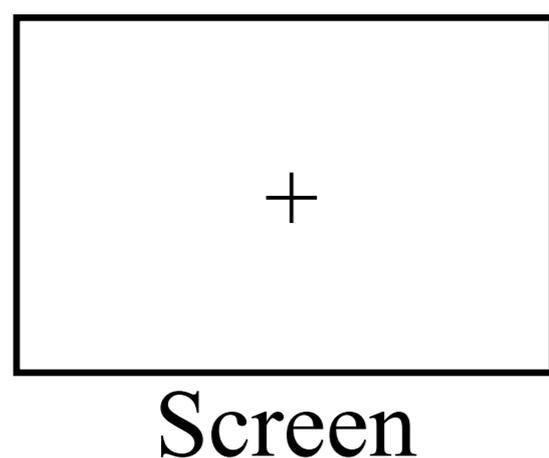
Bed



An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event
- 6 sec ISI, random order

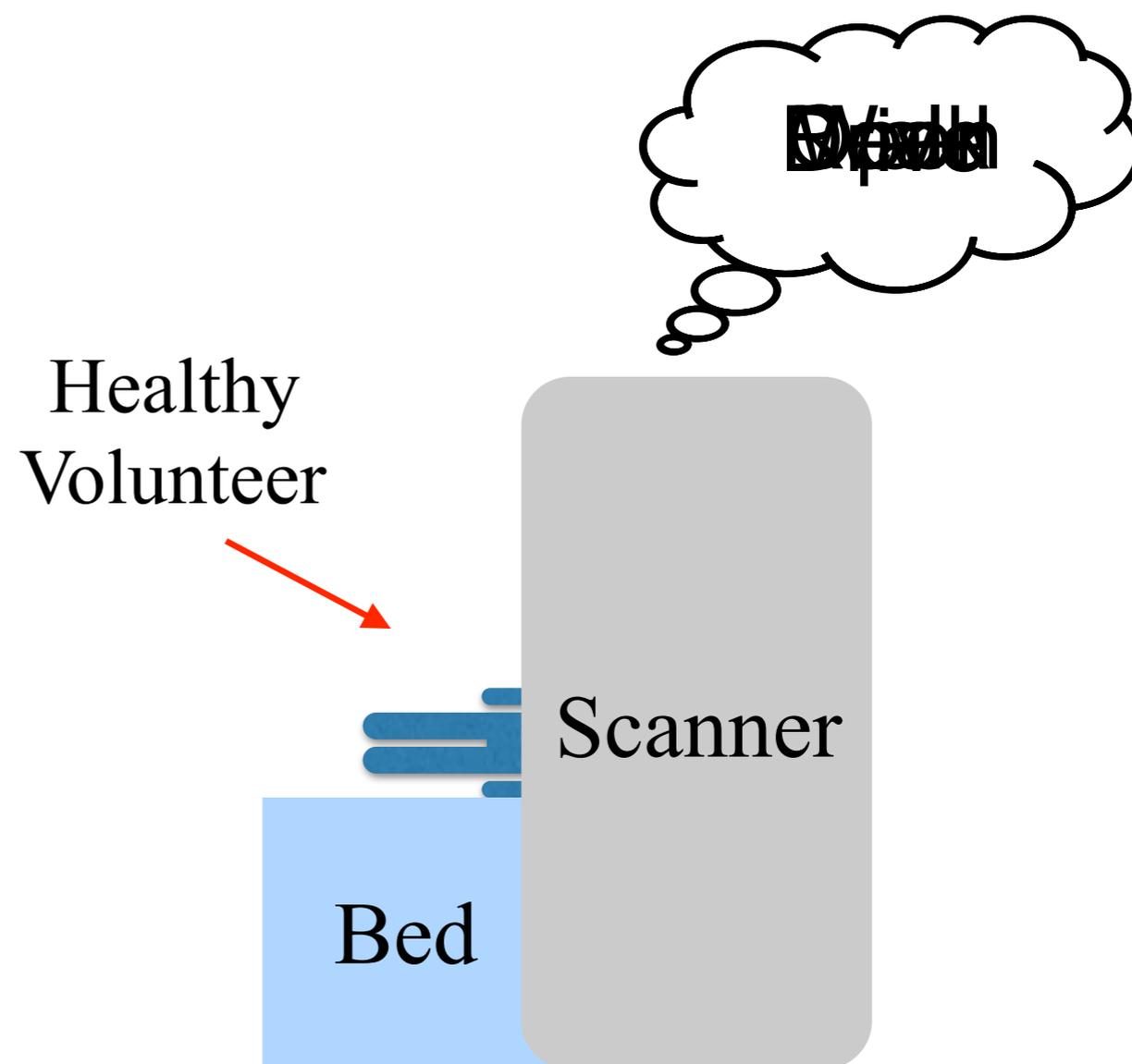
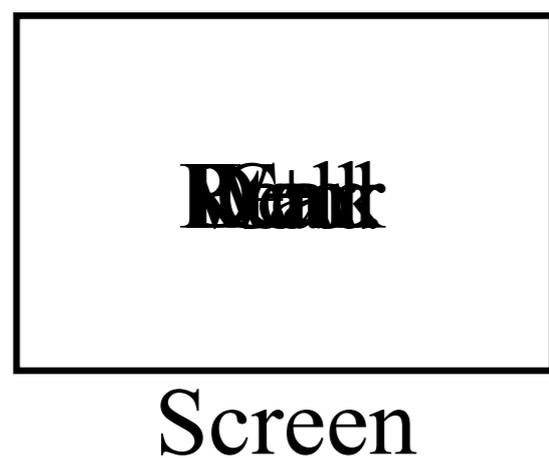




An example experiment

An fMRI adaptation of a classical PET experiment

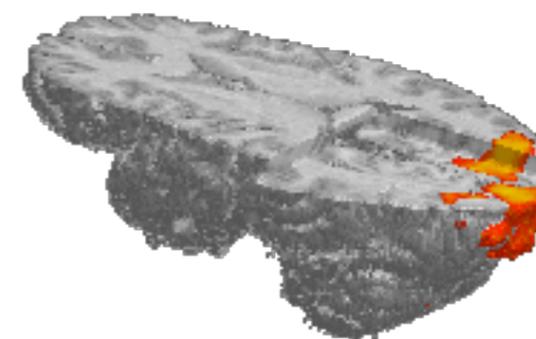
- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event
- 6 sec ISI, random order
- For 24 events of each type





FMRI Modelling and Statistics

- An example experiment
- **Multiple regression (GLM)**
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?

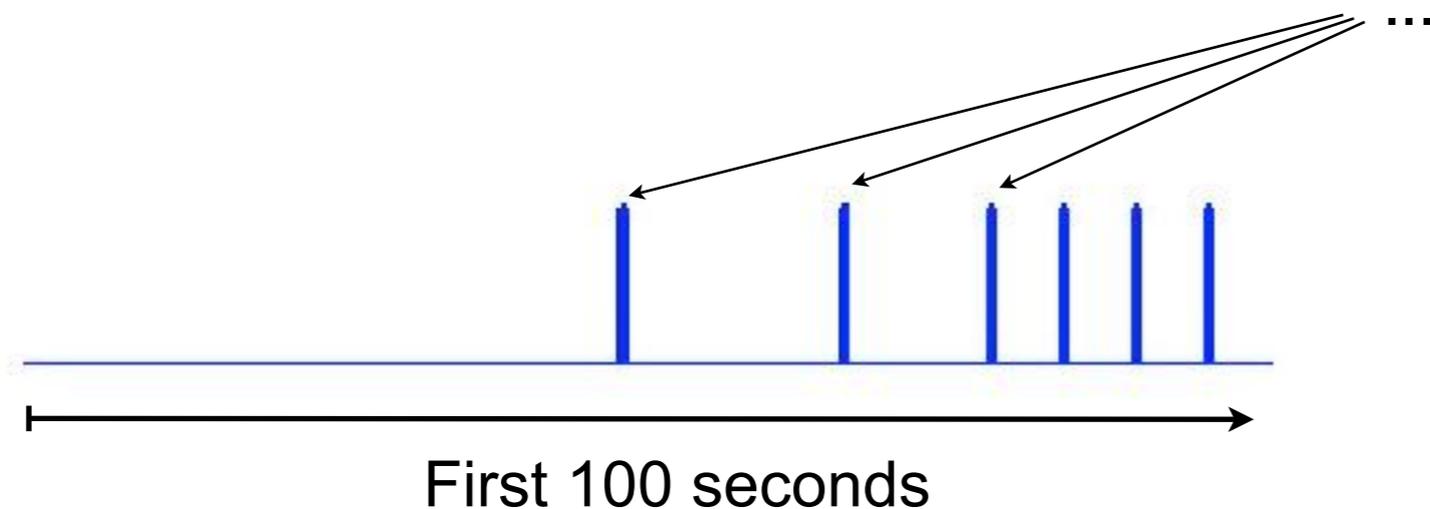


Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?

Stick-function at each occurrence
of a “generation event”



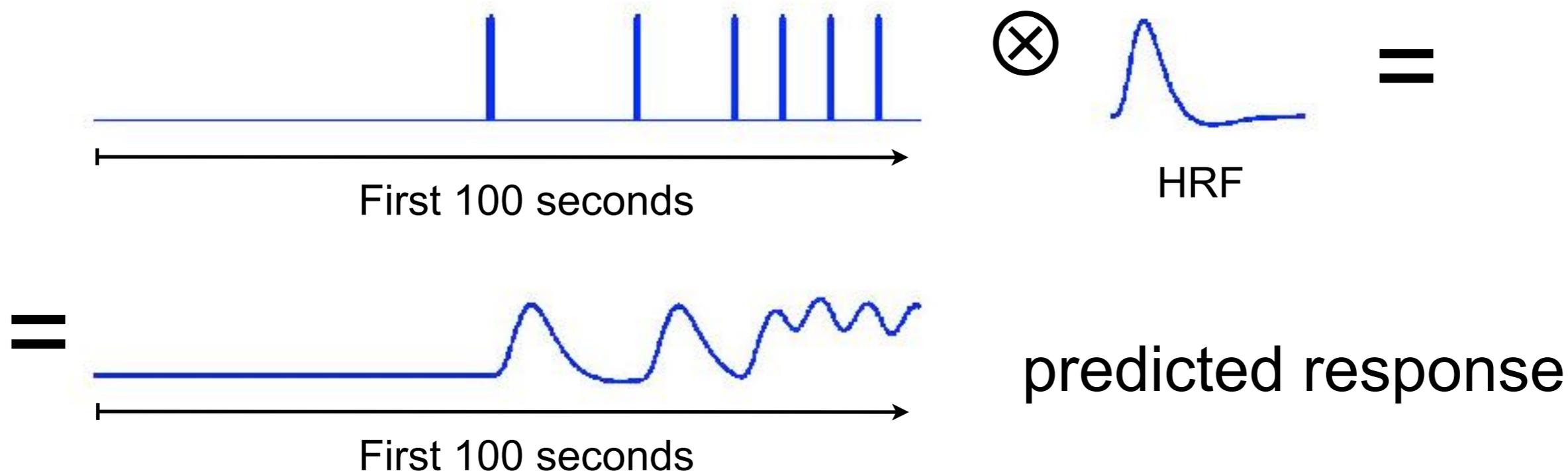
Well, hardly like this...



Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



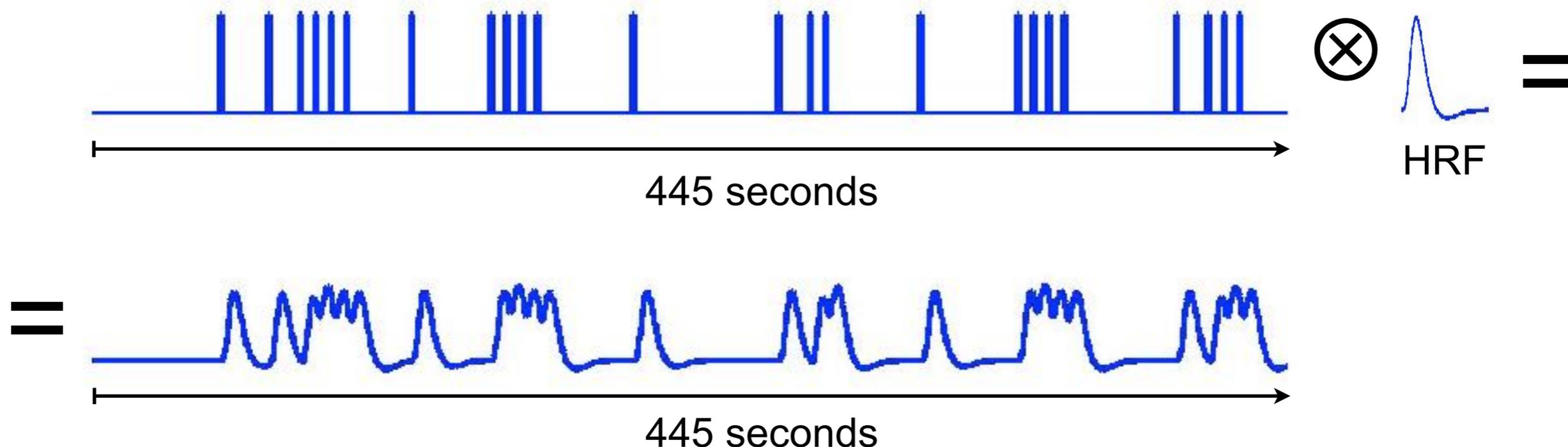
That looks better!



Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



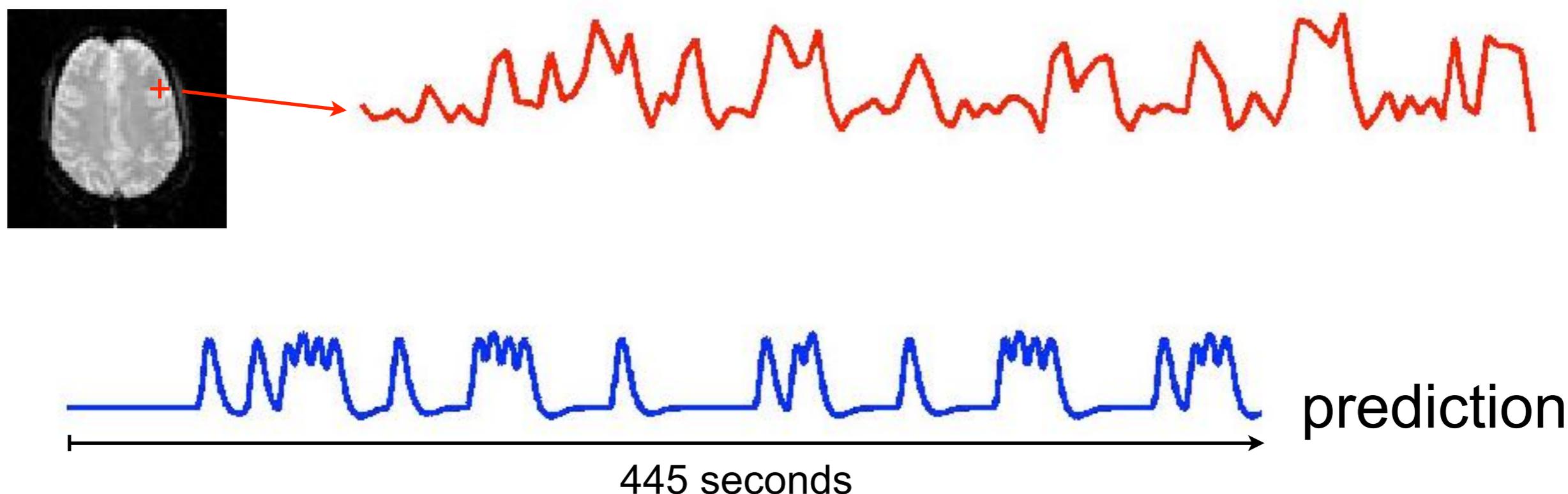
And this is the prediction for the whole time-series



Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



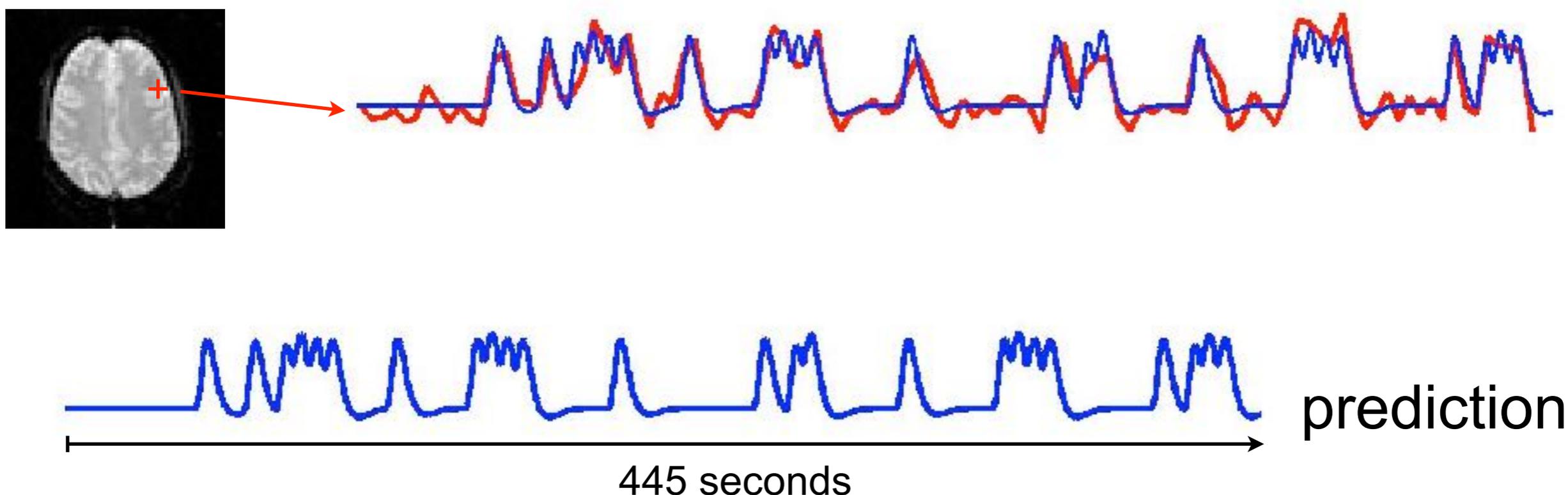
So, if we spot a time-series like this



Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



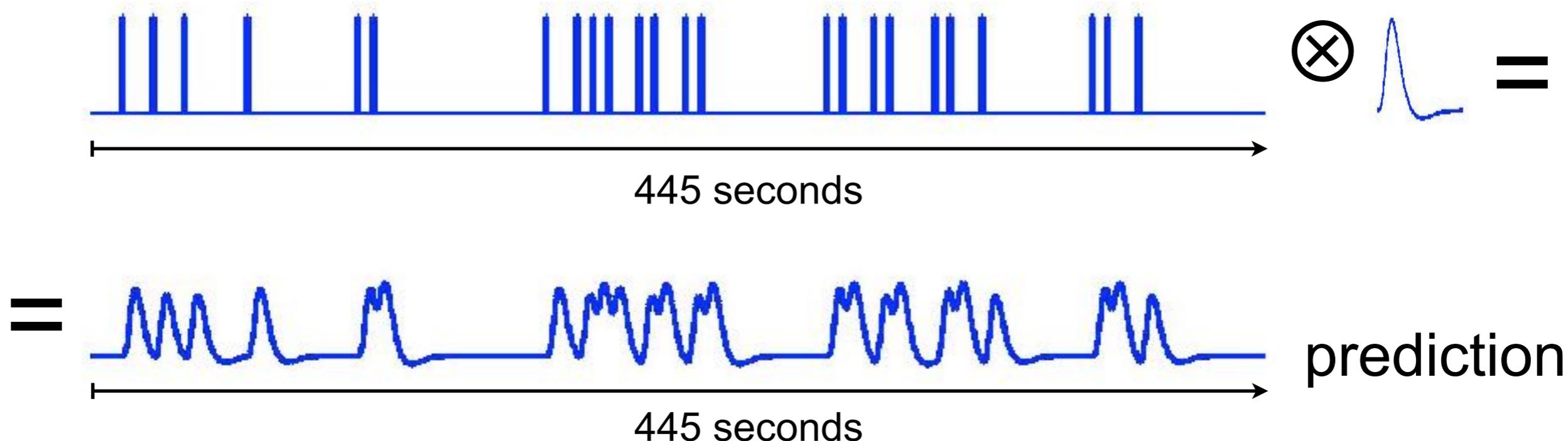
And then check it against our prediction
we can conclude that this pixel is into word generation



Building a model

Our task is now to build a model for that experiment

And we can do the same for the word shadowing events?



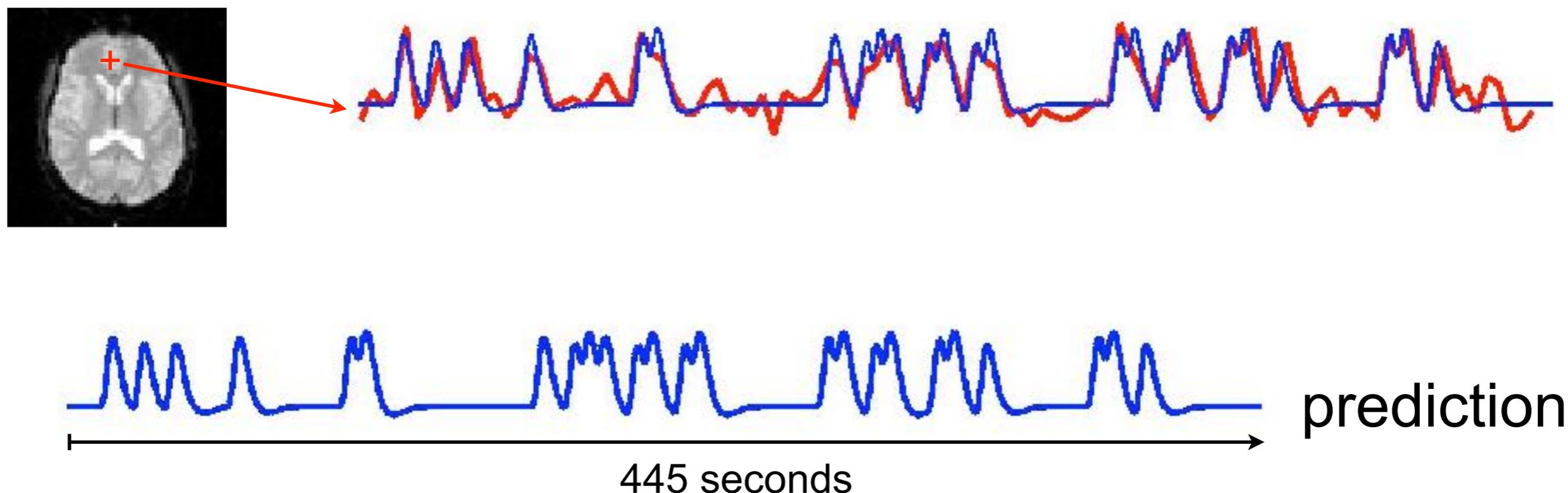
This time we used the onset times for the shadowing events to get the predicted brain response for those



Building a model

Our task is now to build a model for that experiment

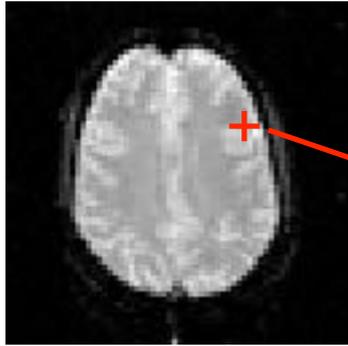
And we can do the same for the word shadowing events?



And we can look for voxels that match that



Formalising it: Multiple regression



Observed data

\approx

β_1

+

β_2

·

Word Generation



Word Shadowing



Predicted responses
“Regressors”

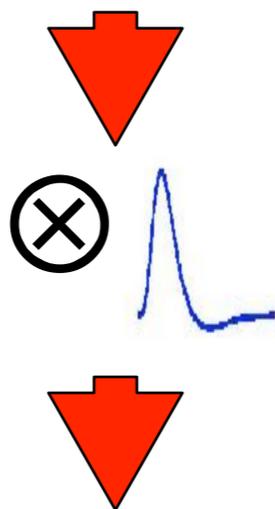
Unknown
“parameters”



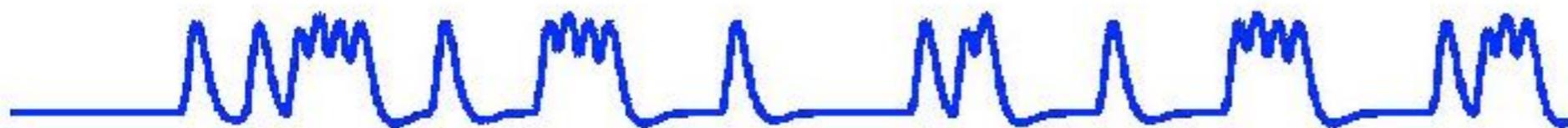
Slight detour: Making regressors



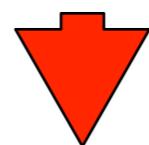
Event timings at “high” resolution



Convolve with HRF



Predictions at “high” resolution



Sub-sample at T_R of experiment



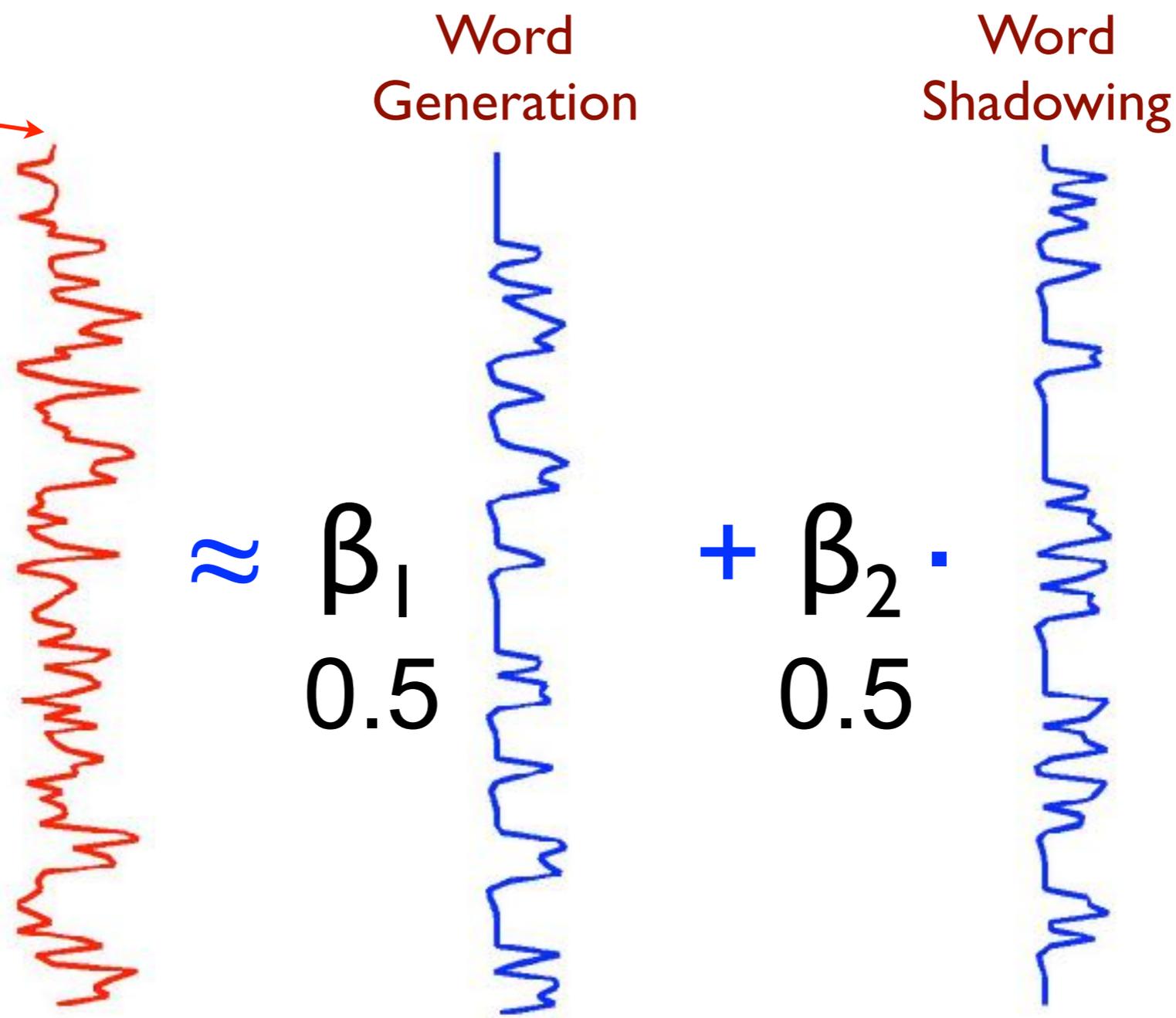
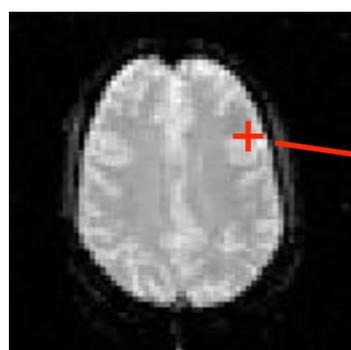
Regressor



Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



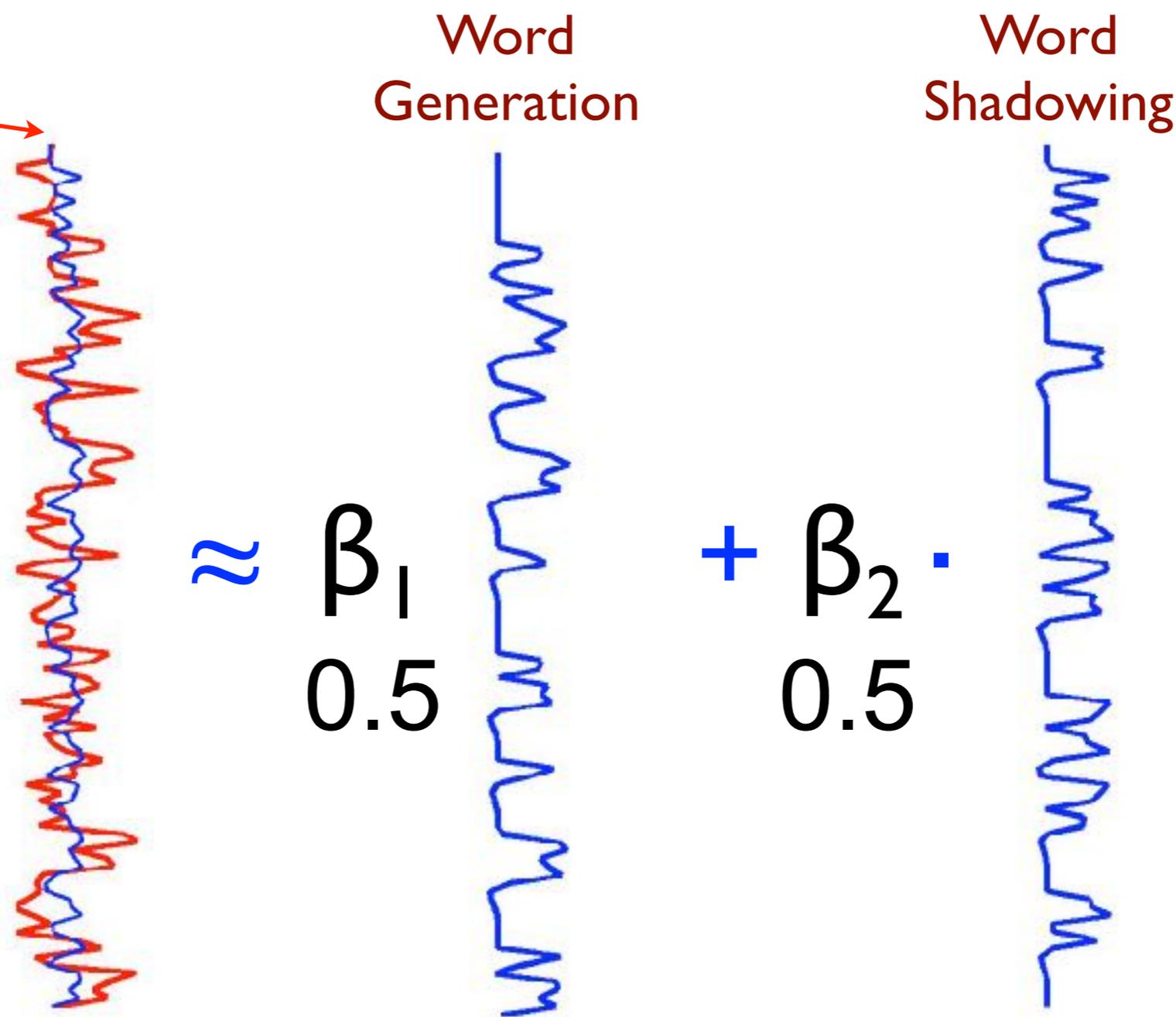
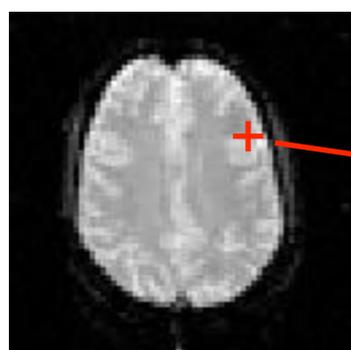
Let's try these parameter values



Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.

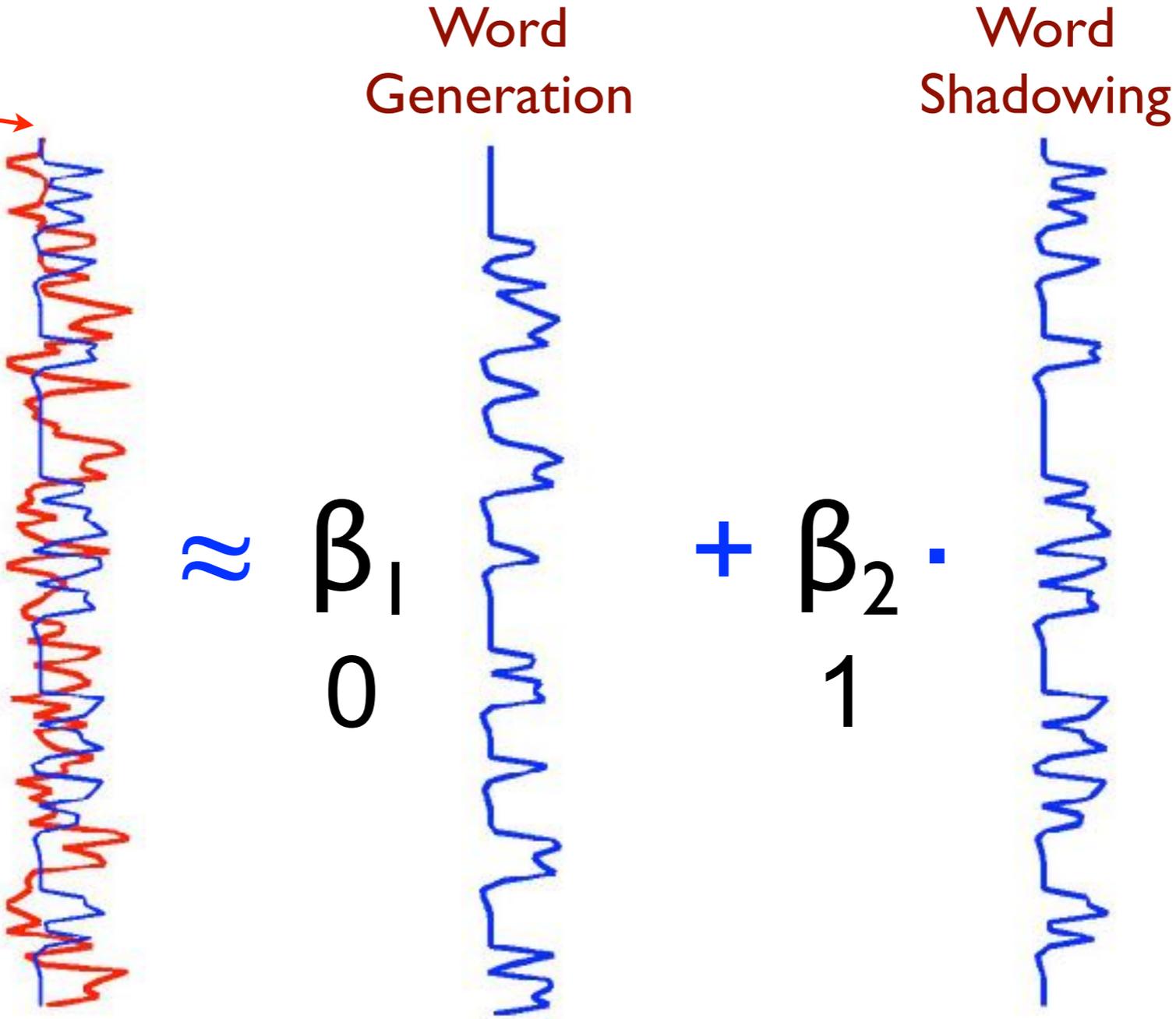
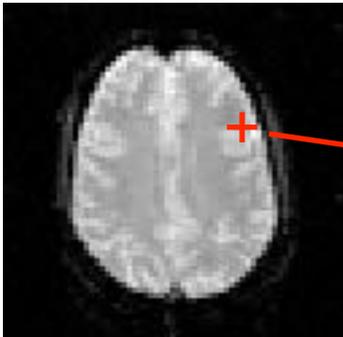




Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



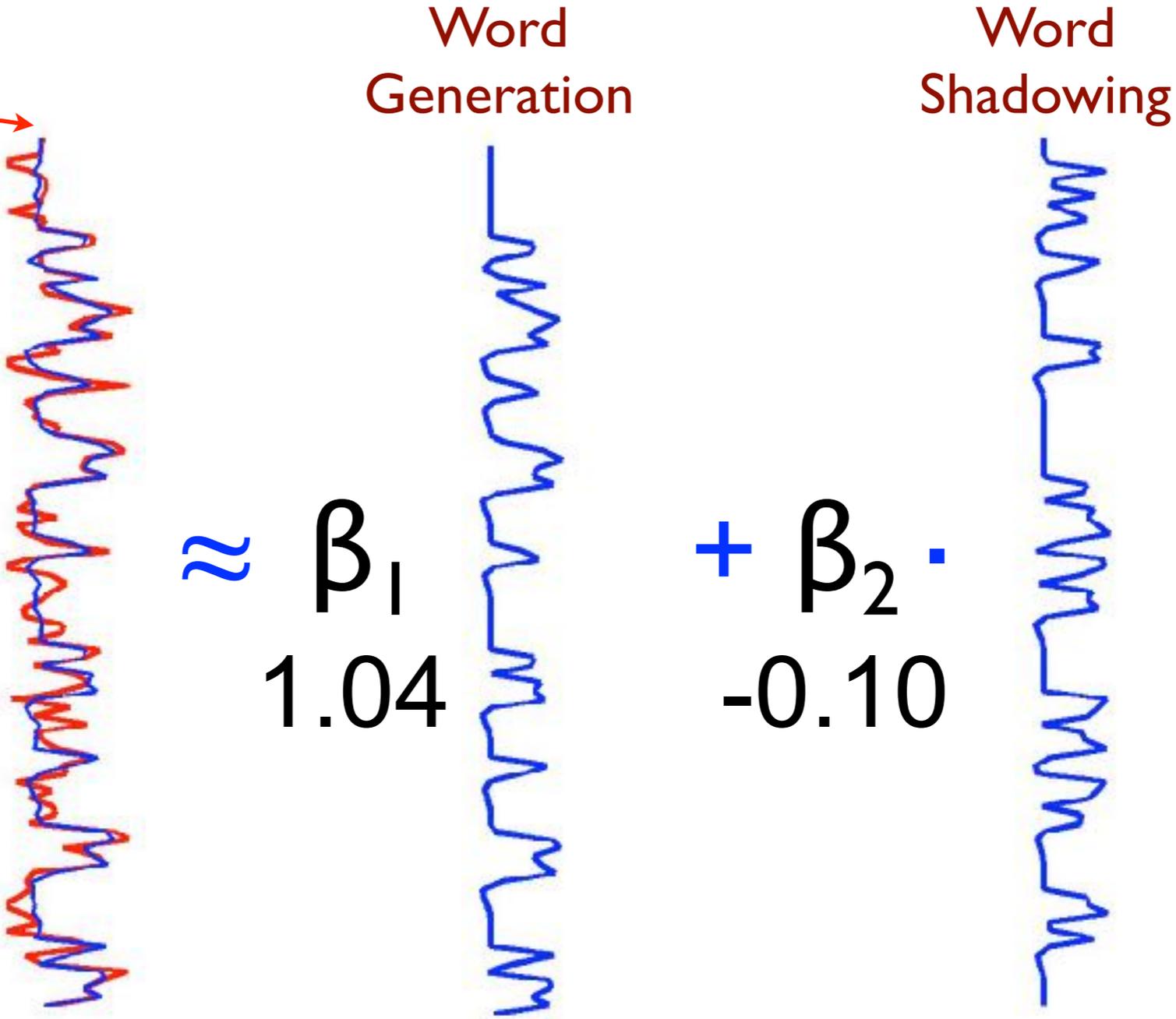
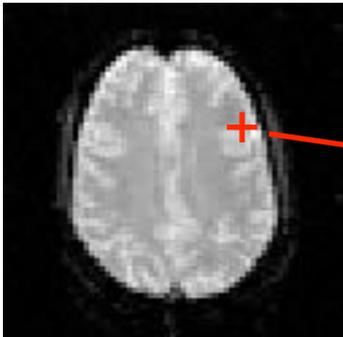
Oh dear,
even worse



Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



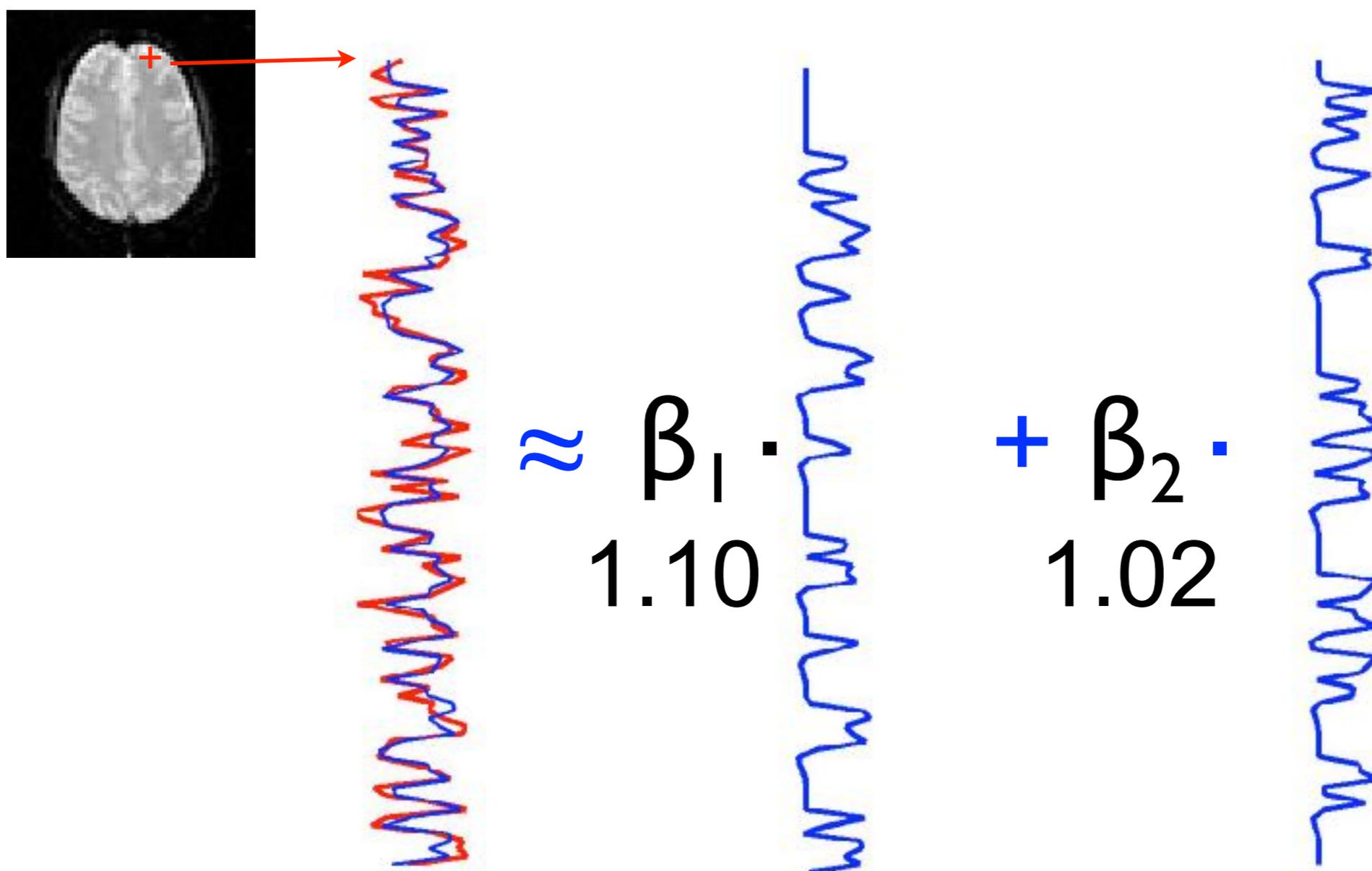
But that looks good



Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data



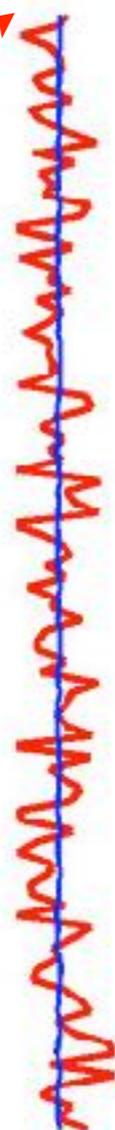
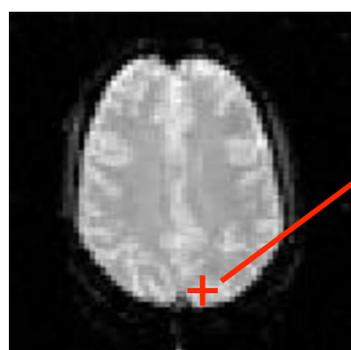
And different voxels yield different parameters



Estimation:

Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data



\approx

$\beta_1 \cdot$

-0.04



$+ \beta_2 \cdot$

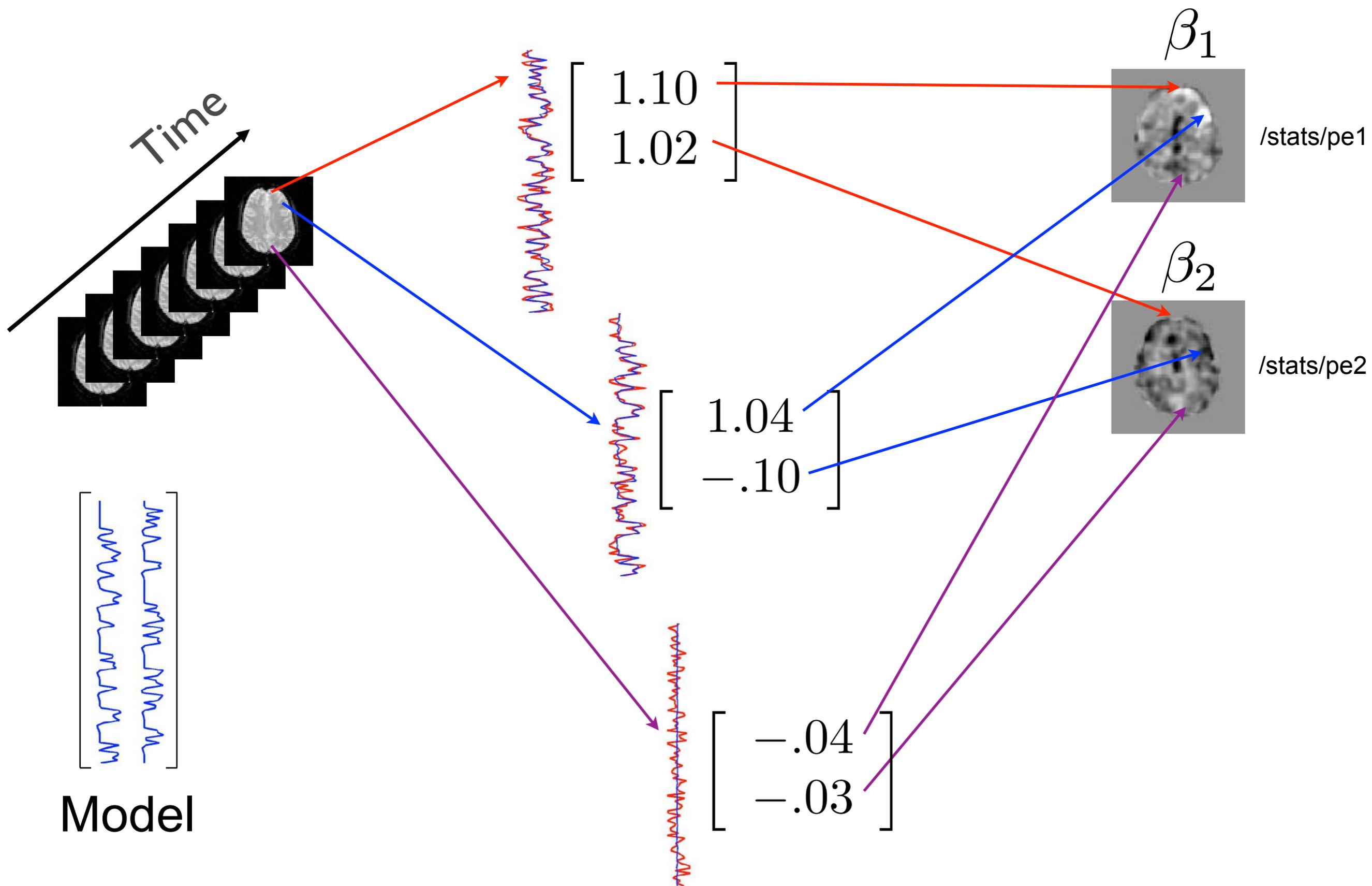
-0.03



And different voxels yield different parameters



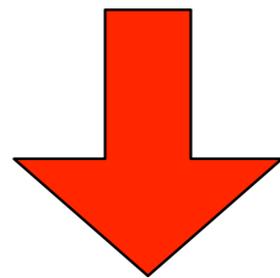
One model to fit them all





And we can also estimate the residual error

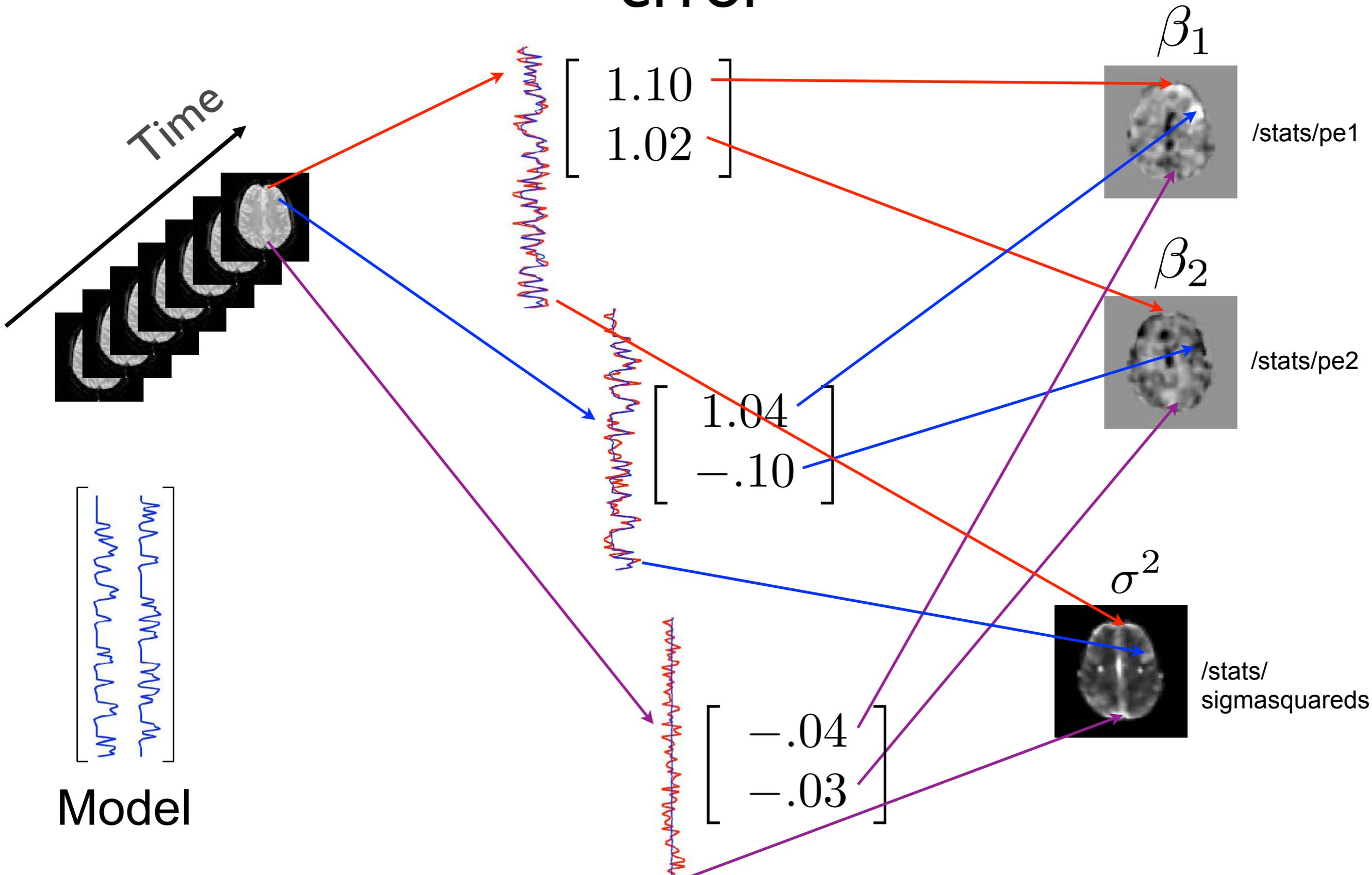
Difference between data and best fit: "Residual error"



Residual errors



And we can also estimate the residual error





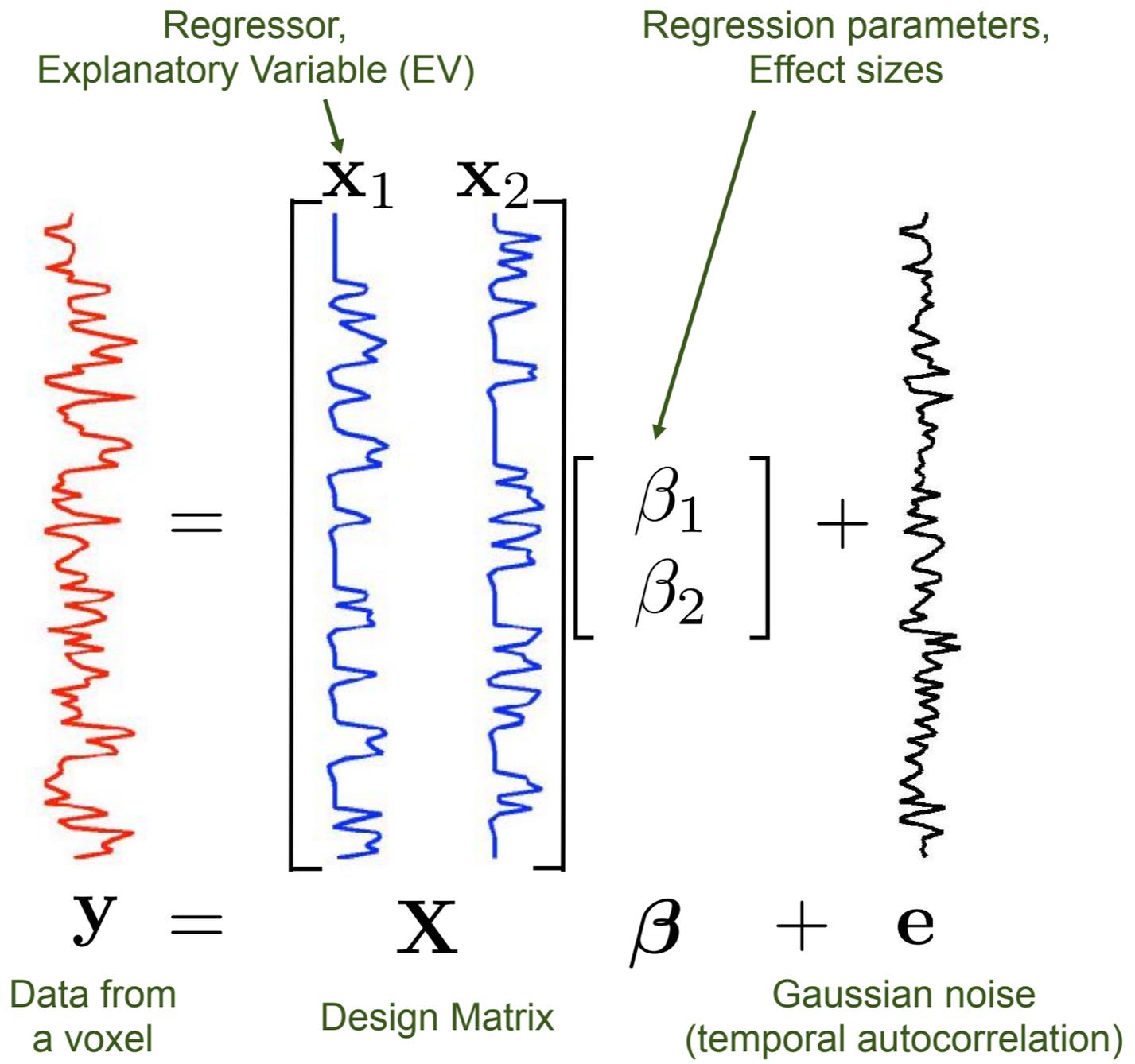
Summary of what we learned so far

- The “Model” consists of a set of “regressors” i.e. tentative time series that we expect to see as a response to our stimulus
- The model typically consists of our stimulus functions convolved by the HRF
- The estimation entails finding the parameter values such that the linear combination of regressors “best” fits the data
- Every voxel has its own unique parameter values, that is how a single model can fit so many different time series
- We can also get an estimate of the error through the “residuals”



General Linear Model (GLM)

- This is placed into the General Linear Model (GLM) framework

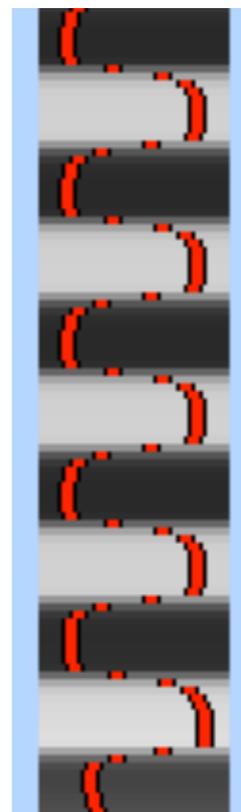




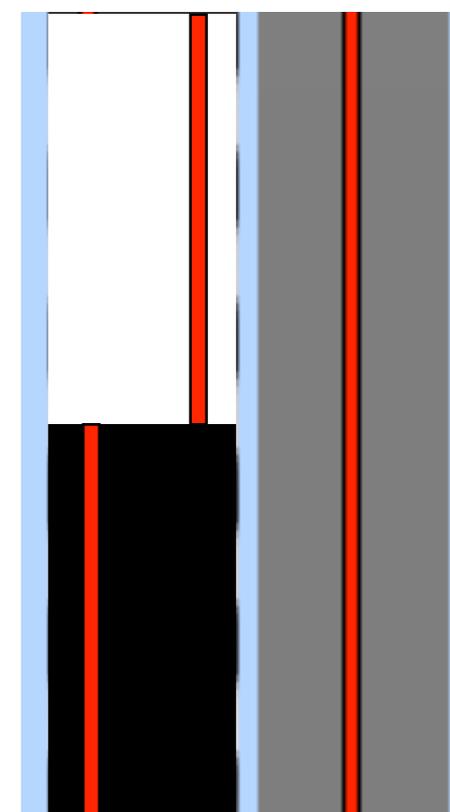
“Demeaning” and the GLM

- The mean value is uninteresting in an fMRI session
- There are two equivalent options:
 1. remove the mean from the data and don't model it
 2. put a term into the model to account for the mean

option #1



option #2



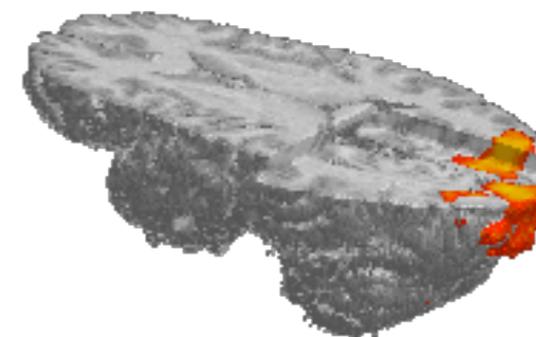
In FSL we use option #1 for first-level analyses and #2 for higher-level analyses

A consequence is that the baseline condition in first-level analysis is **NOT** explicitly modelled (in FSL)



FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- **T and F Contrasts**
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





t-contrasts

- A contrast of parameter estimates (COPE) is a linear combination of PEs:

$$[1 \ 0]: \text{COPE} = 1 \times \hat{\beta}_1 + 0 \times \hat{\beta}_2 = \hat{\beta}_1$$

$$[1 \ -1]: \text{COPE} = 1 \times \hat{\beta}_1 + -1 \times \hat{\beta}_2 = \hat{\beta}_1 - \hat{\beta}_2$$

- Test null hypothesis that $\text{COPE}=0$

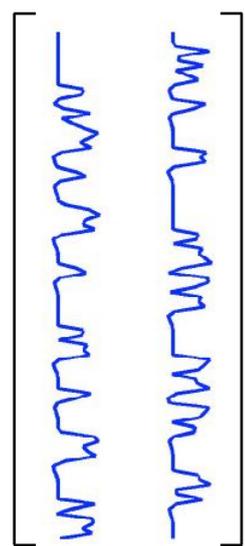
t-statistic: $t = \frac{\text{COPE}}{\text{std}(\text{COPE})}$



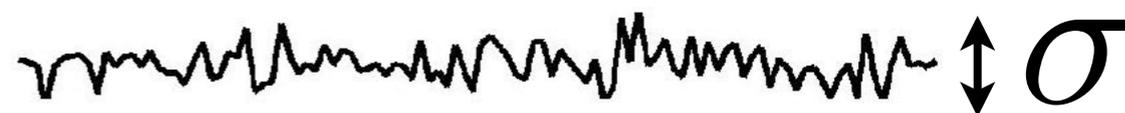
t-contrasts

$$t = \frac{COPE}{std(COPE)}$$

Depends on



[1 0]



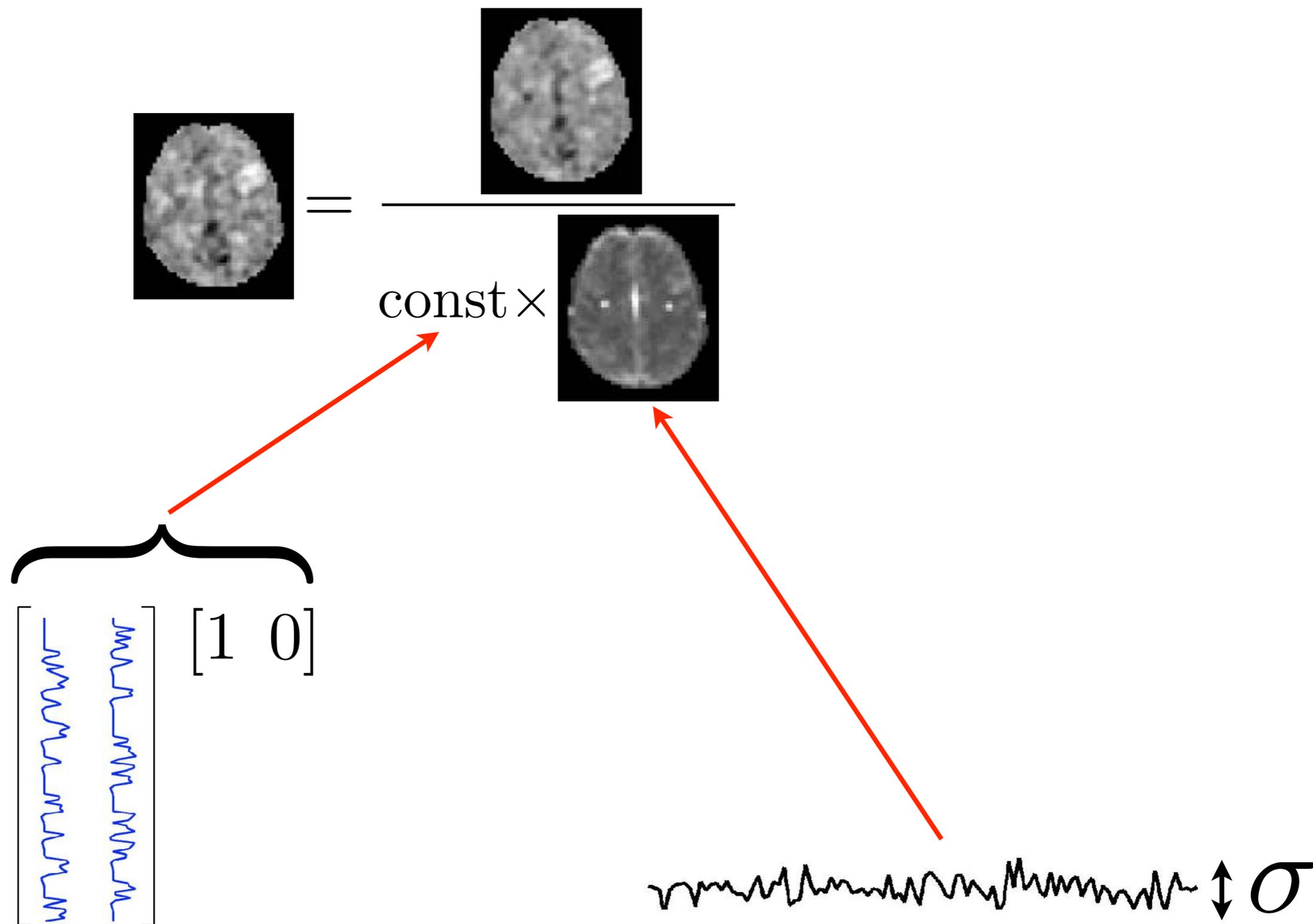
The Model

,the Contrast

and the Residual Error



t-contrasts



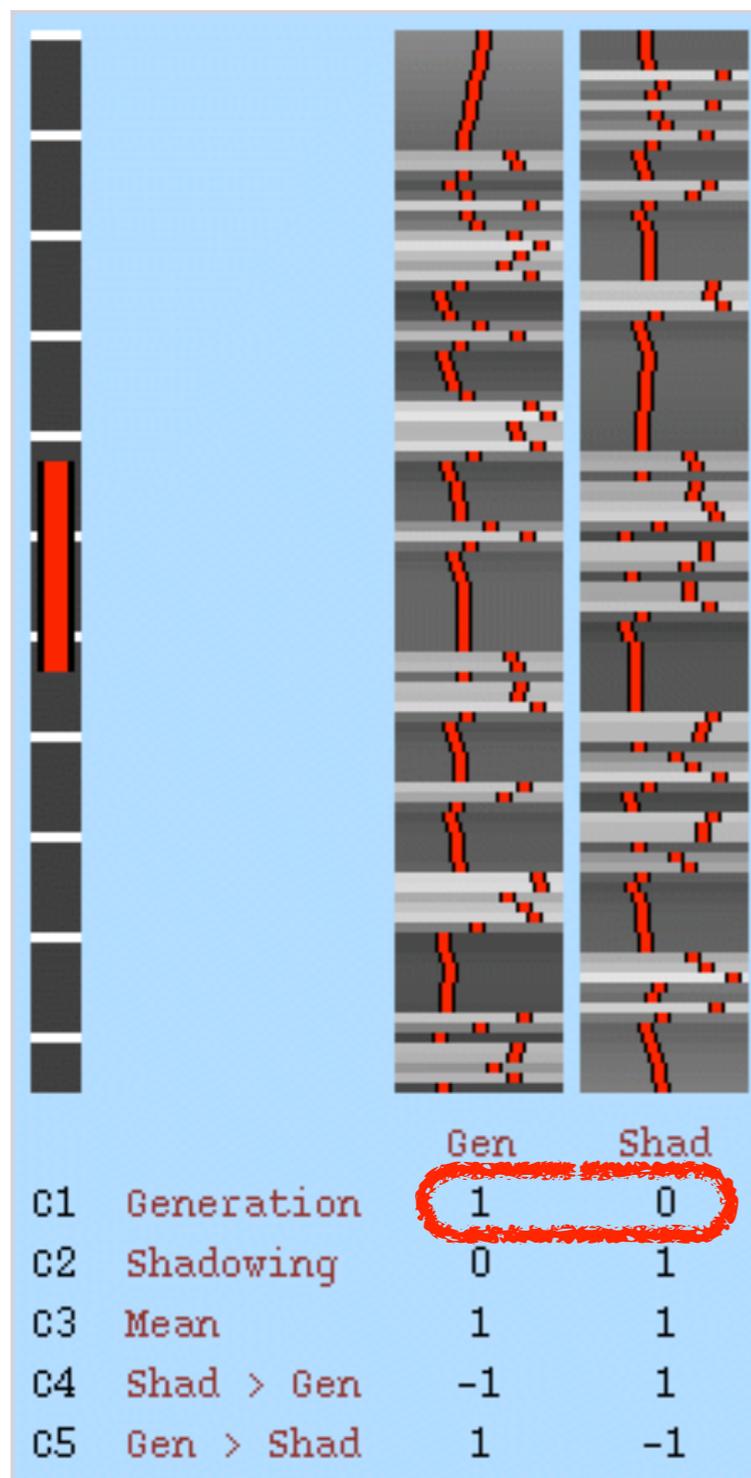
The Model & the Contrast

and the Residual Error



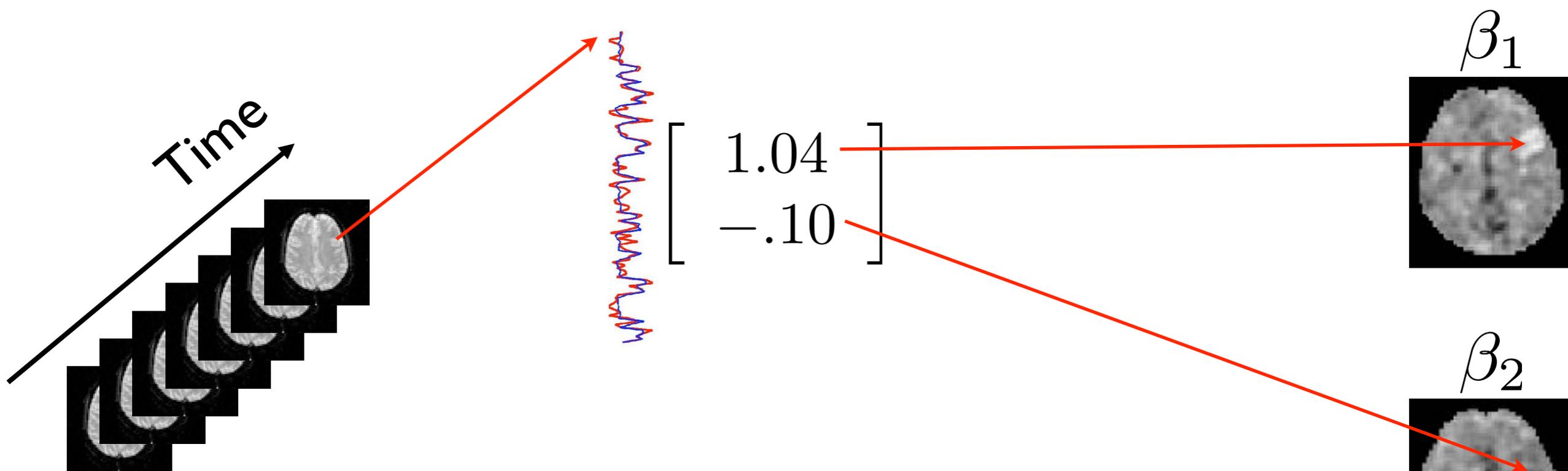
t-contrasts

- $[1\ 0]$: EV1 only (i.e. Generation vs rest)
- $[0\ 1]$: EV2 only (i.e. Shadowing vs rest)



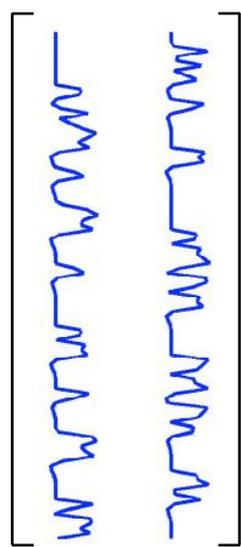


t-contrasts



Contrast weight vector: $[1 \ 0]$

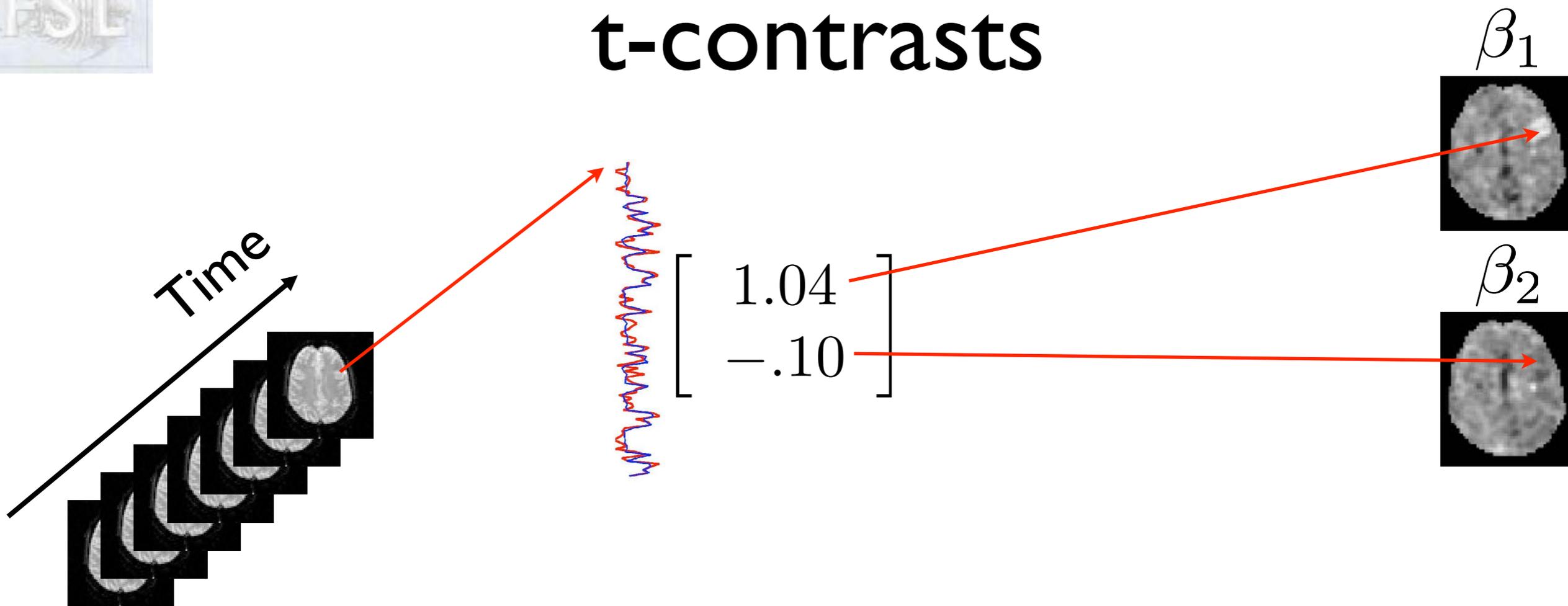
Asks the question: Where do we need this regressor to model the data, i.e. what parts of the brain are used when seeing nouns and generating related verbs?



Model

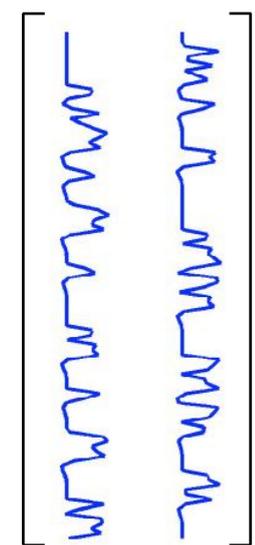


t-contrasts



Contrast weight vector: $[1 \ 0]$

$$\text{COPE} = 1 \times 1.04 + 0 \times -0.10 = 1.04$$

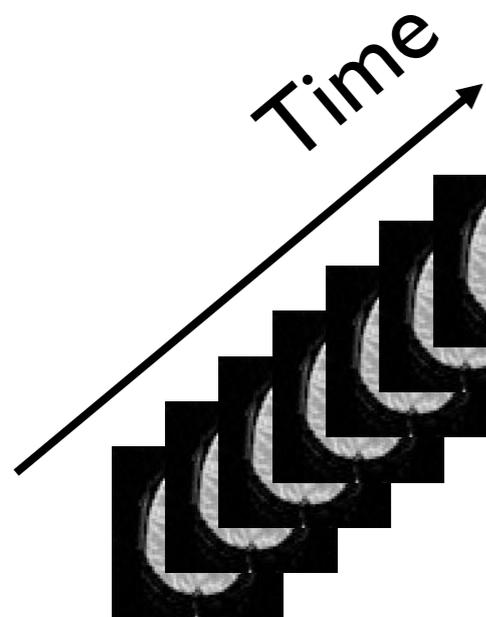


Model

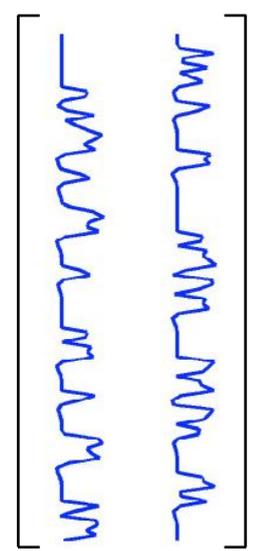
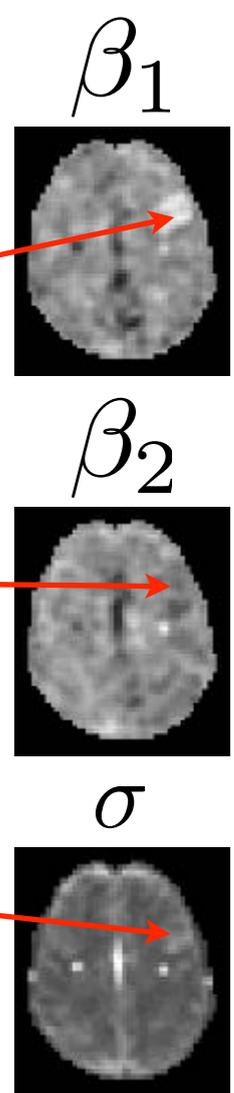
$$\text{COPE} = \text{[Brain Slice]} = \beta_1$$



t-contrasts



$$\begin{bmatrix} 1.04 \\ -.10 \end{bmatrix}$$



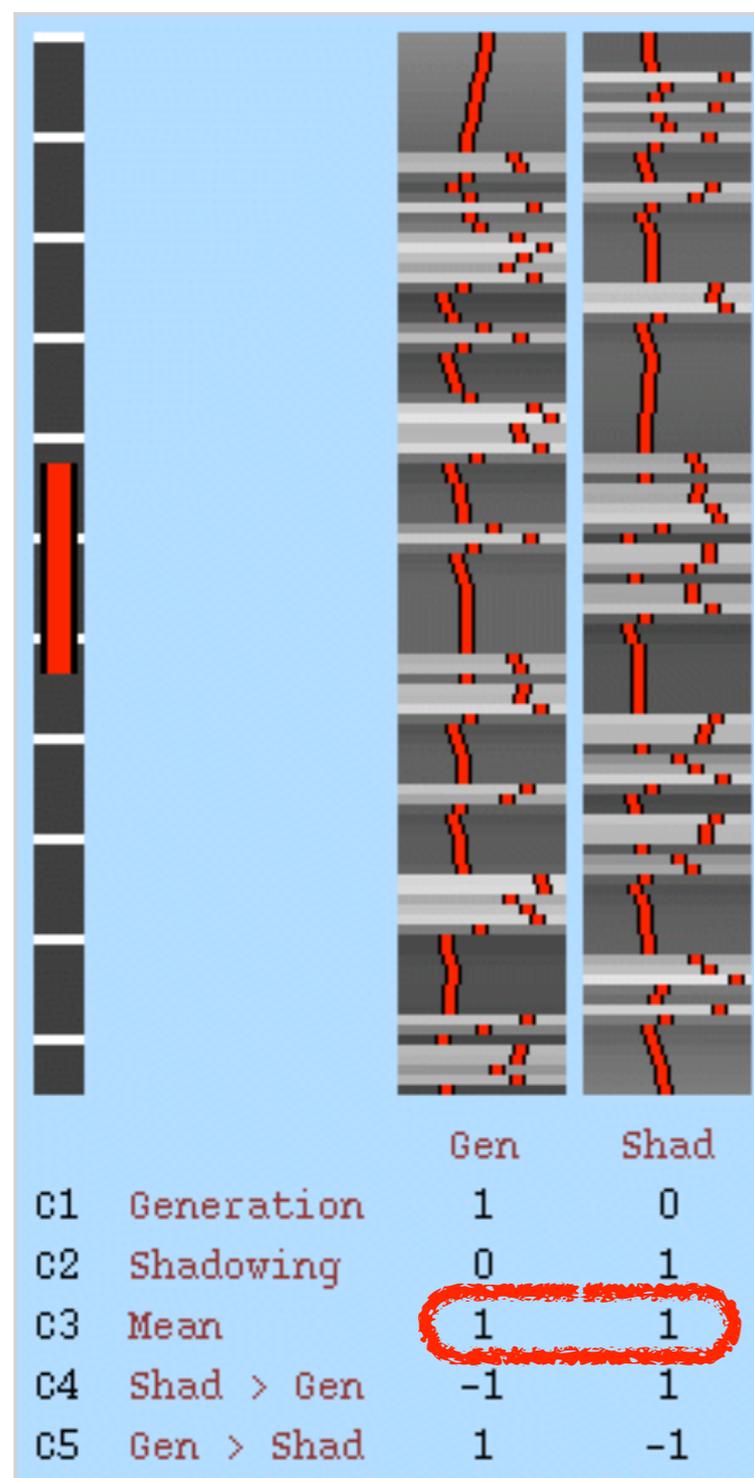
Model

$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{[Brain Slice]}}{\text{[Brain Slice]}} = \text{[Brain Slice]}$$



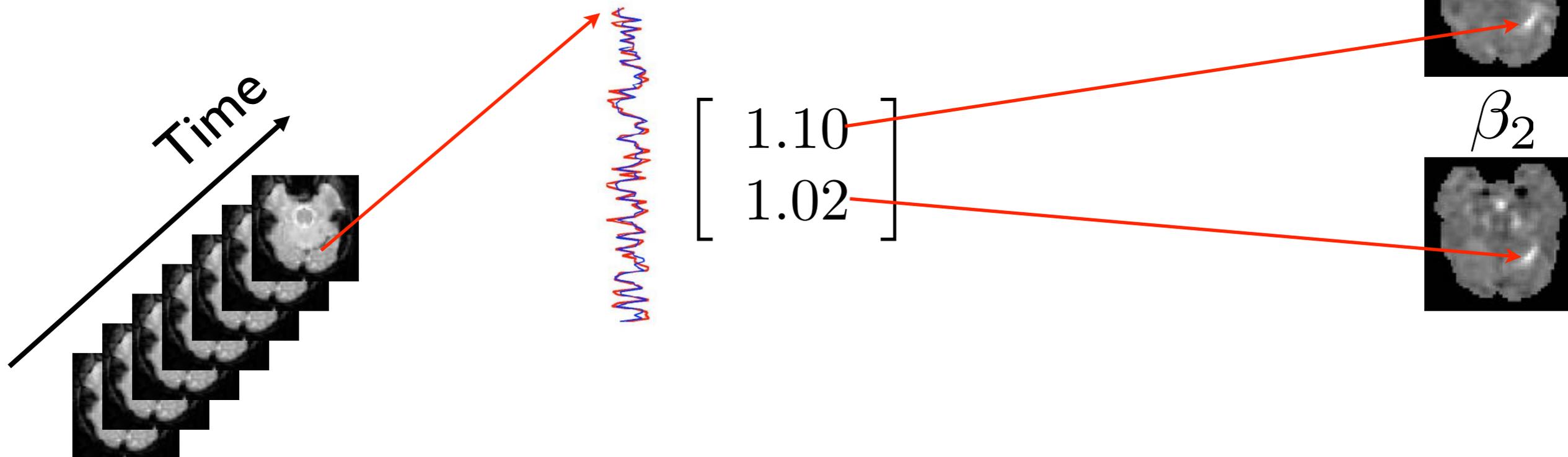
t-contrasts

- $[1\ 0]$: EV1 only (i.e. Generation vs rest)
- $[0\ 1]$: EV2 only (i.e. Shadowing vs rest)
- $[1\ 1]$: EV1 + EV2 (Mean activation)



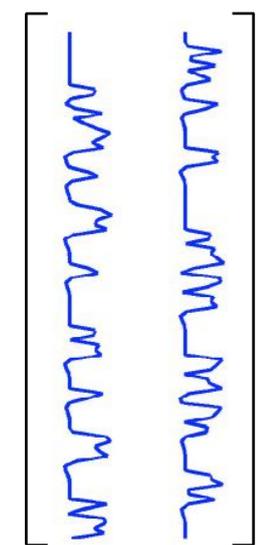


t-contrasts



Contrast weight vector: $[1 \ 1]$

$$\text{COPE} = 1 \times 1.10 + 1 \times 1.02 = 2.12$$



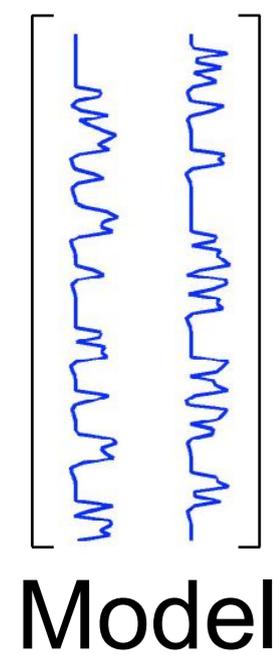
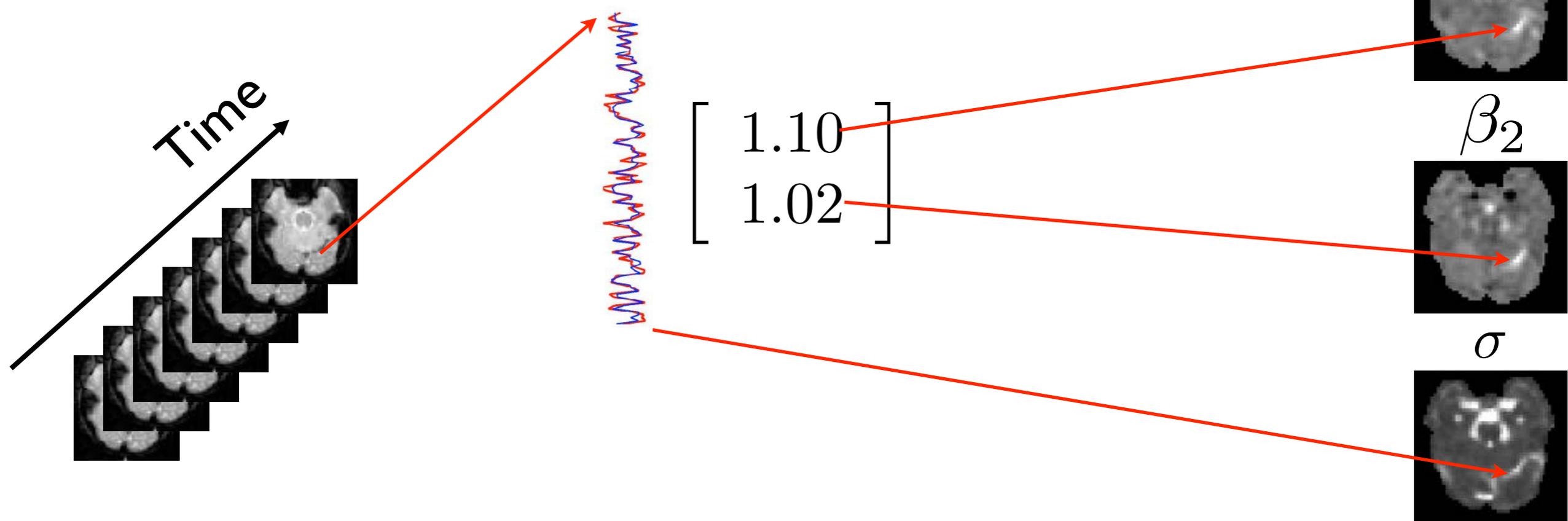
Model

$$\text{COPE} = \text{Image} = \beta_1 + \beta_2$$

The equation shows the Contrast of Parameters Estimate (COPE) as a linear combination of contrast images. The first image is the COPE itself, which is equal to the sum of the two contrast images, β_1 and β_2 .



t-contrasts

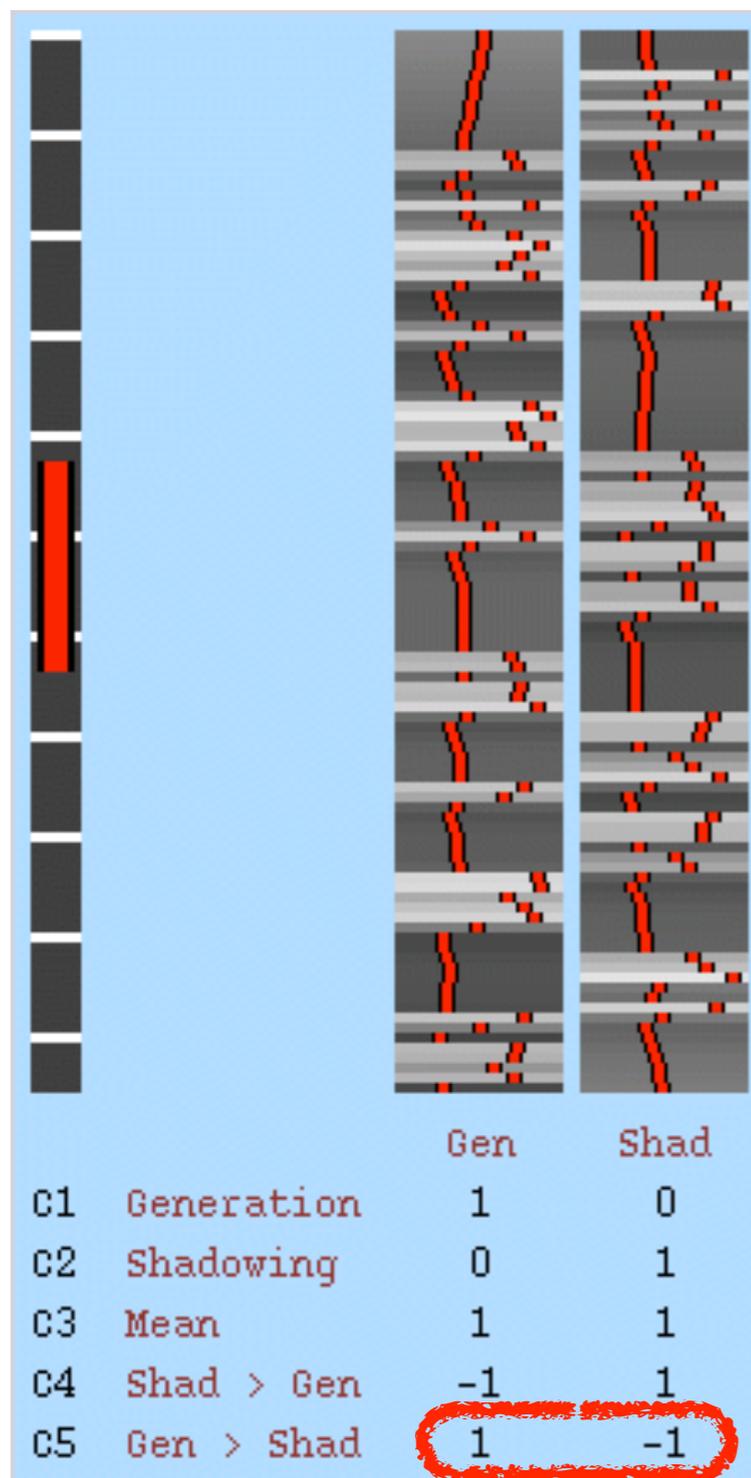


$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{Image 1}}{\text{Image 2}} = \text{Image 3}$$



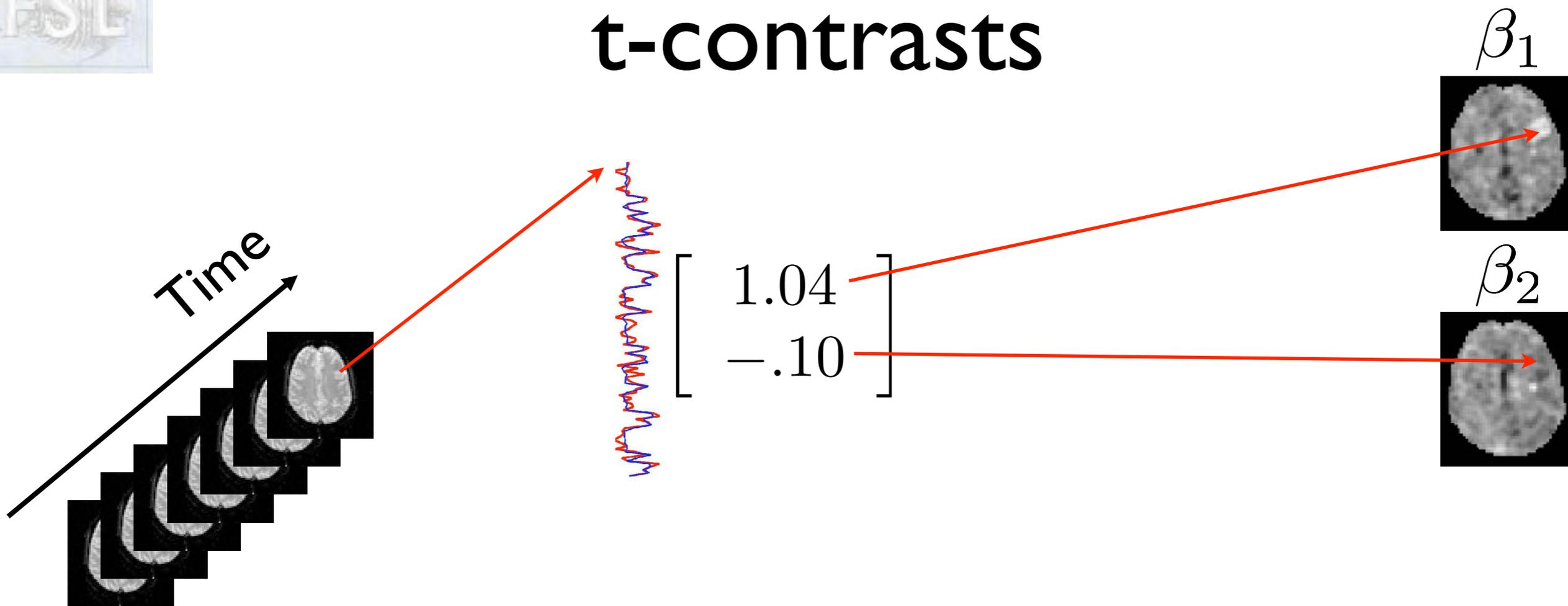
t-contrasts

- $[1\ 0]$: EV1 only (i.e. Generation vs rest)
- $[0\ 1]$: EV2 only (i.e. Shadowing vs rest)
- $[1\ 1]$: EV1 + EV2 (Mean activation)
- $[-1\ 1]$: EV2 - EV1 (More activated by Shadowing than Generation)
- $[1\ -1]$: EV1 - EV2 (More activated by Generation than Shadowing (*t*-tests are directional))



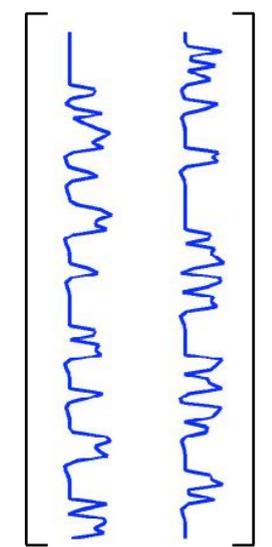


t-contrasts



Contrast weight vector: $[1 \quad -1]$

$$\text{COPE} = 1 \times 1.04 - 1 \times -0.10 = 1.14$$



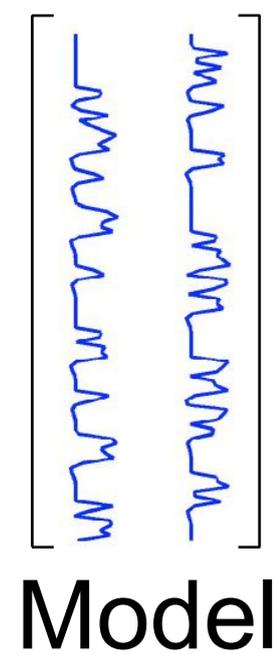
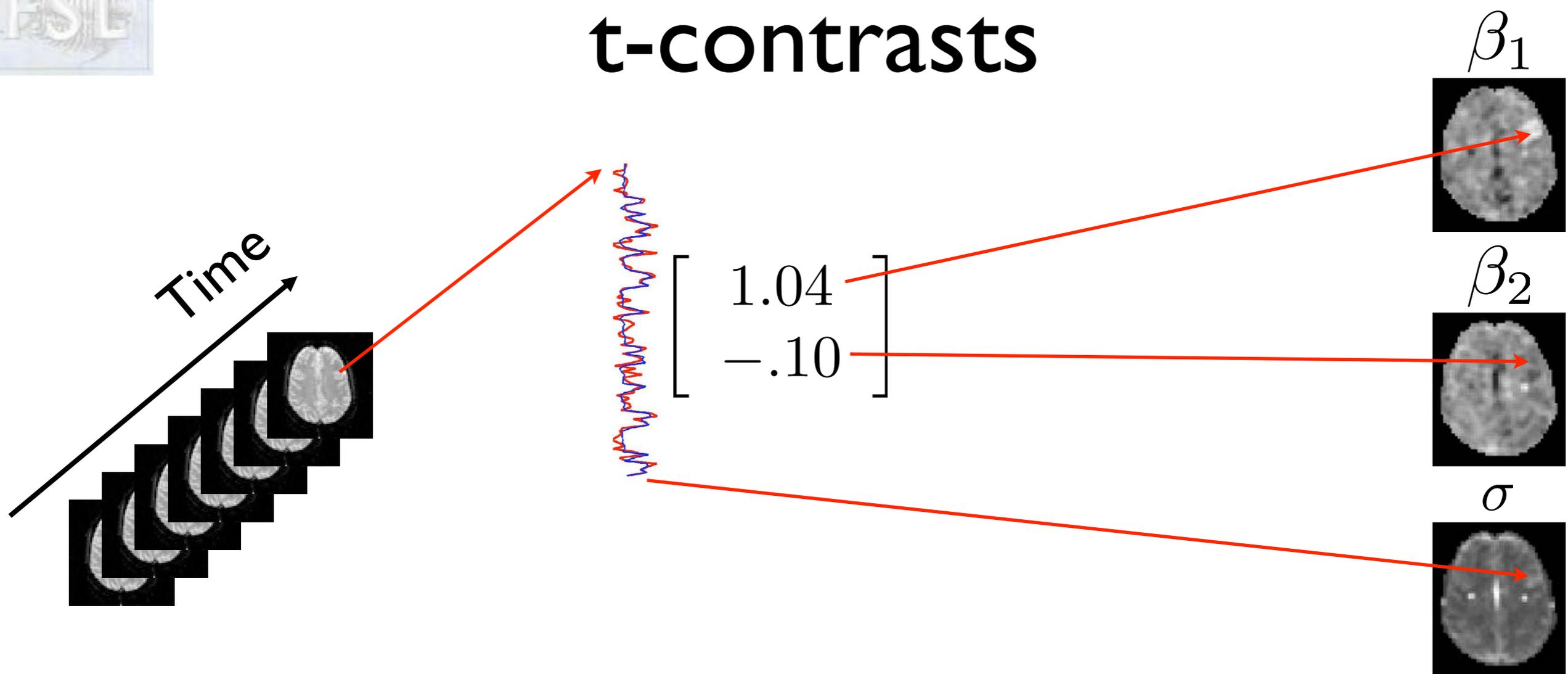
Model

$$\text{COPE} = \text{Image} = \beta_1 - \beta_2$$

The equation shows the COPE result as a difference of two contrast images, β_1 and β_2 .



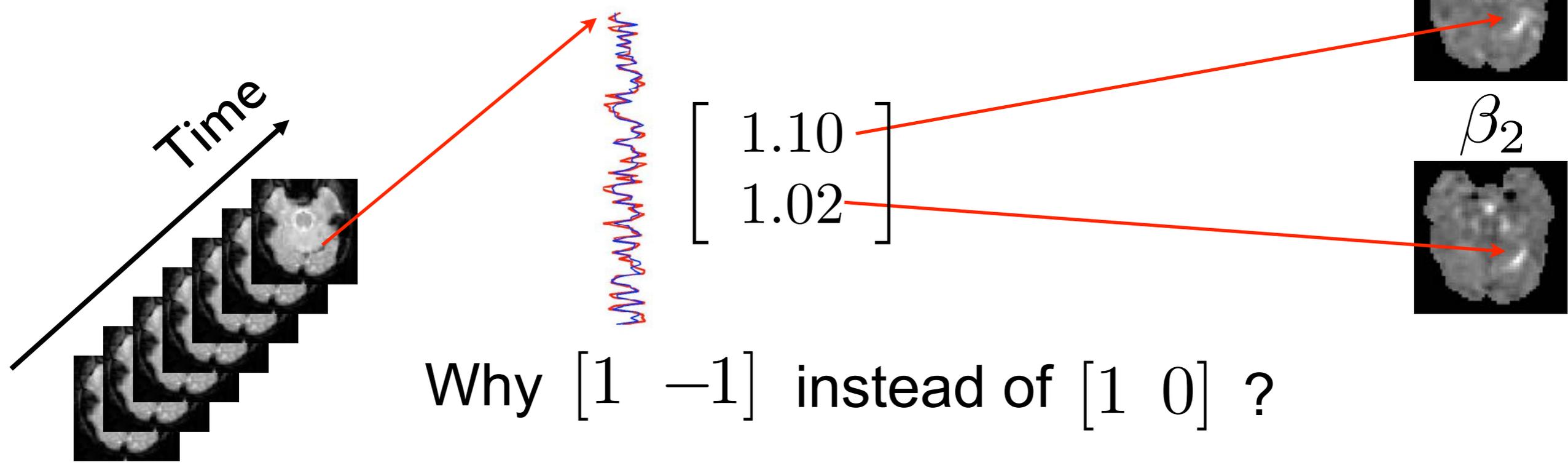
t-contrasts



$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{Brain Slice 1}}{\text{Brain Slice 2}} = \text{Brain Slice 3}$$



t-contrasts



$$\frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}}{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1.10 \\ 1.02 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

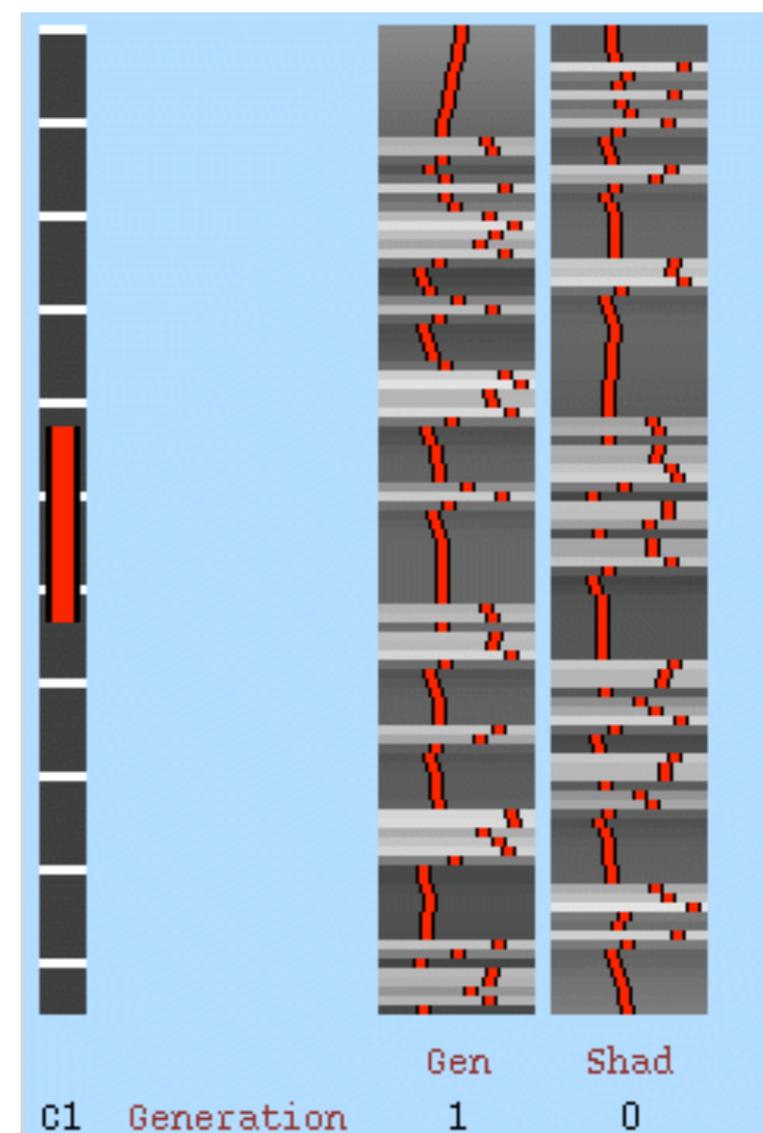
$$\frac{\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}}{\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1.10 \\ 1.02 \end{bmatrix}} = \frac{\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}}{\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1.10 \\ 1.02 \end{bmatrix}} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$



F-contrasts

We have two conditions:
Word Generation and Shadowing

We want to know:
Is there an activation to any condition?



First we ask: Is there activation to Generation?

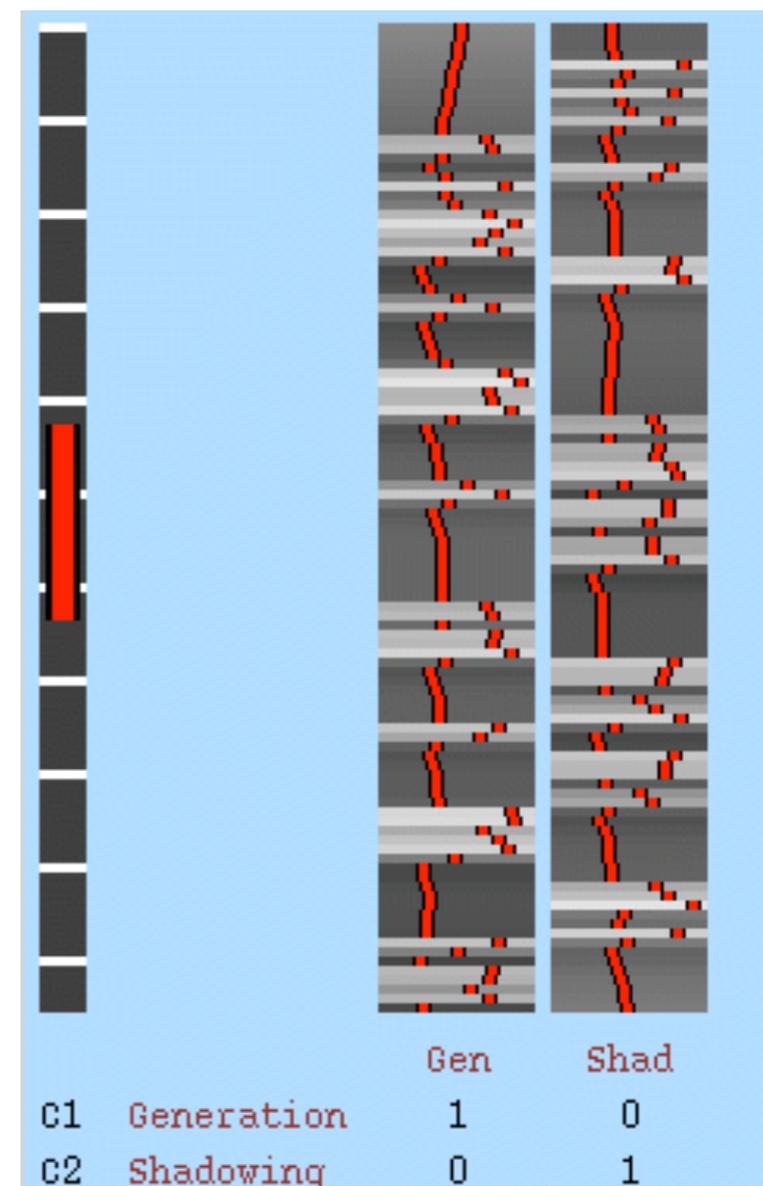
$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$



F-contrasts

We have two conditions:
Word Generation and Shadowing

We want to know:
Is there an activation to any condition?



Then we ask: Is there activation to Shadowing?

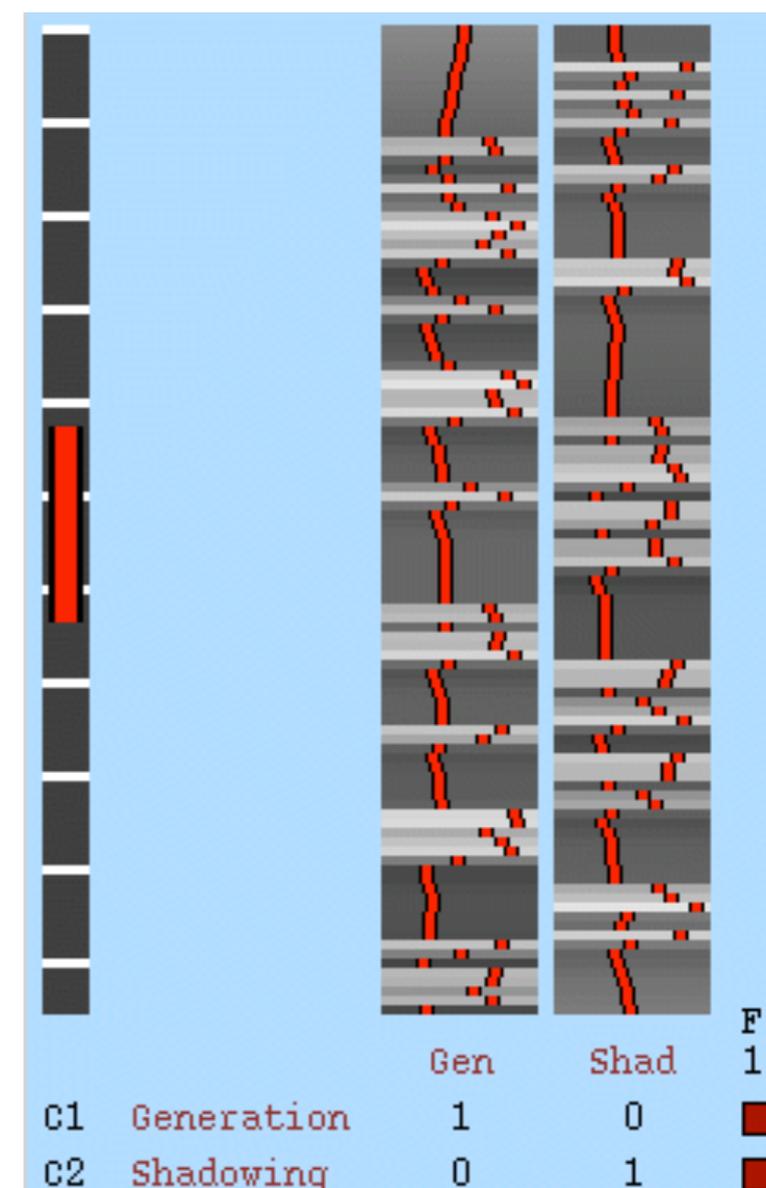
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



F-contrasts

We have two conditions:
Word Generation and Shadowing

We want to know:
Is there an activation to any condition?



Then we add the OR

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

EVs **Contrasts & F-tests**

Setup contrasts & F-tests for

Contrasts F-tests

		Title	EV1	EV2	F1
OC1	<input checked="" type="checkbox"/>	Generation	<input type="text" value="1"/>	<input type="text" value="0"/>	<input checked="" type="checkbox"/>
OC2	<input checked="" type="checkbox"/>	Shadowing	<input type="text" value="0"/>	<input type="text" value="2"/>	<input checked="" type="checkbox"/>



F-contrasts

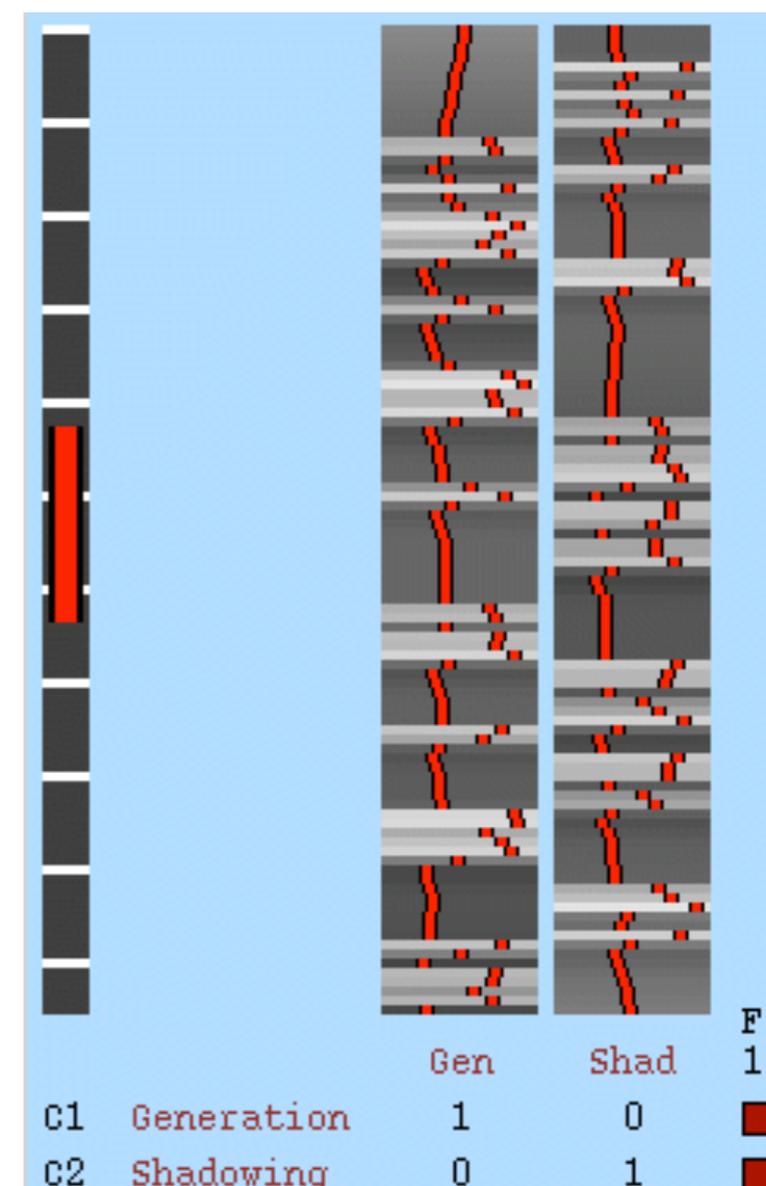
We have two conditions:
Word Generation and Shadowing

We want to know:
Is there an activation to any condition?

Is there an activation to any condition?

Is equivalent to:

Does any regressor explain the
variance in the data?

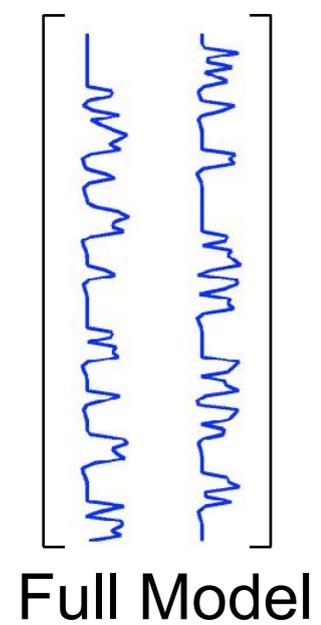


Then we add the OR

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



F-contrasts

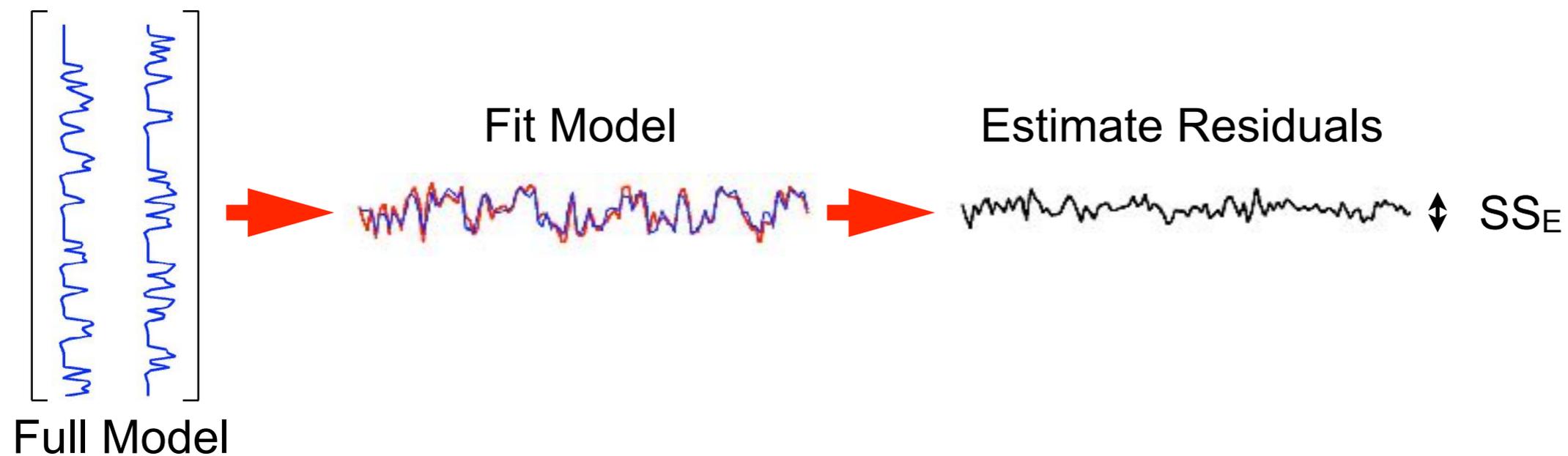


&





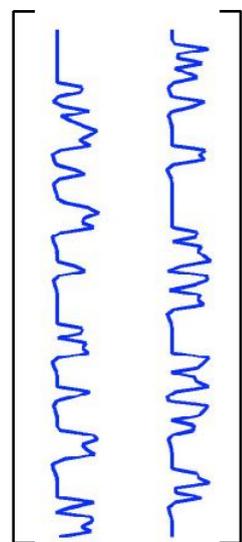
F-contrasts





F-contrasts

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



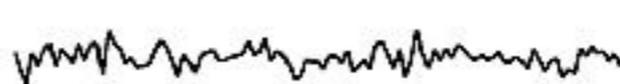
Full Model



Fit Model

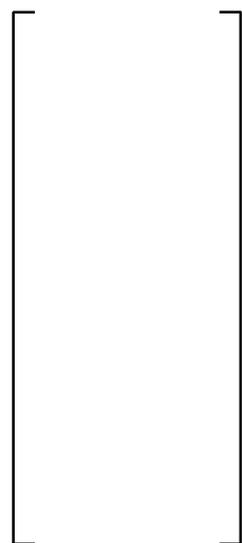


Estimate Residuals

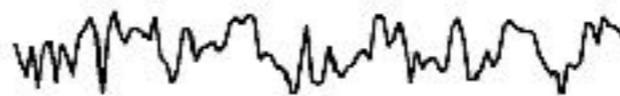
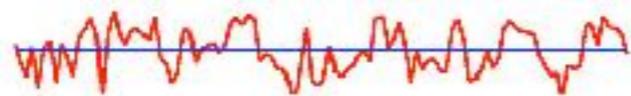


↕ SS_E

$$F = \frac{SS_R - SS_E}{SS_E} = \frac{\updownarrow - \updownarrow}{\updownarrow}$$



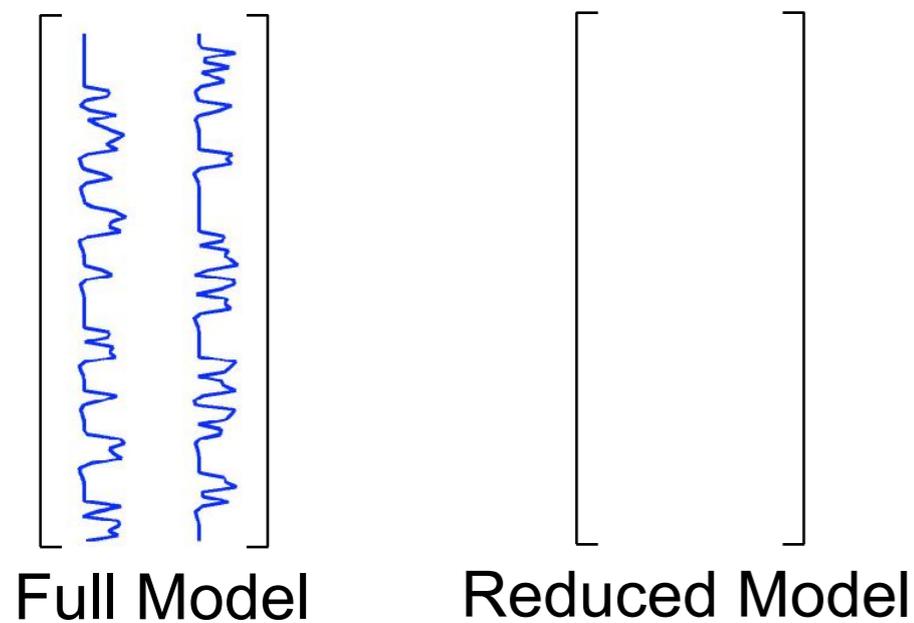
Reduced Model



↕ SS_R



F-contrasts



$$F = \frac{SS_R - SS_E}{SS_E} = \frac{\text{Brain}_1 - \text{Brain}_2}{\text{Brain}_3} = \frac{\text{Brain}_4}{\text{Brain}_5} = \text{Brain}_6$$

The equation illustrates the calculation of the F-contrast. It starts with the formula $F = \frac{SS_R - SS_E}{SS_E}$. This is then visualized as a ratio of two brain images: the numerator is the difference between two brain images (Brain₁ - Brain₂), and the denominator is a single brain image (Brain₃). This is further simplified to a ratio of two brain images (Brain₄ / Brain₅), where Brain₄ is the difference image and Brain₅ is the reference image. The final result is a single brain image (Brain₆) representing the F-contrast.

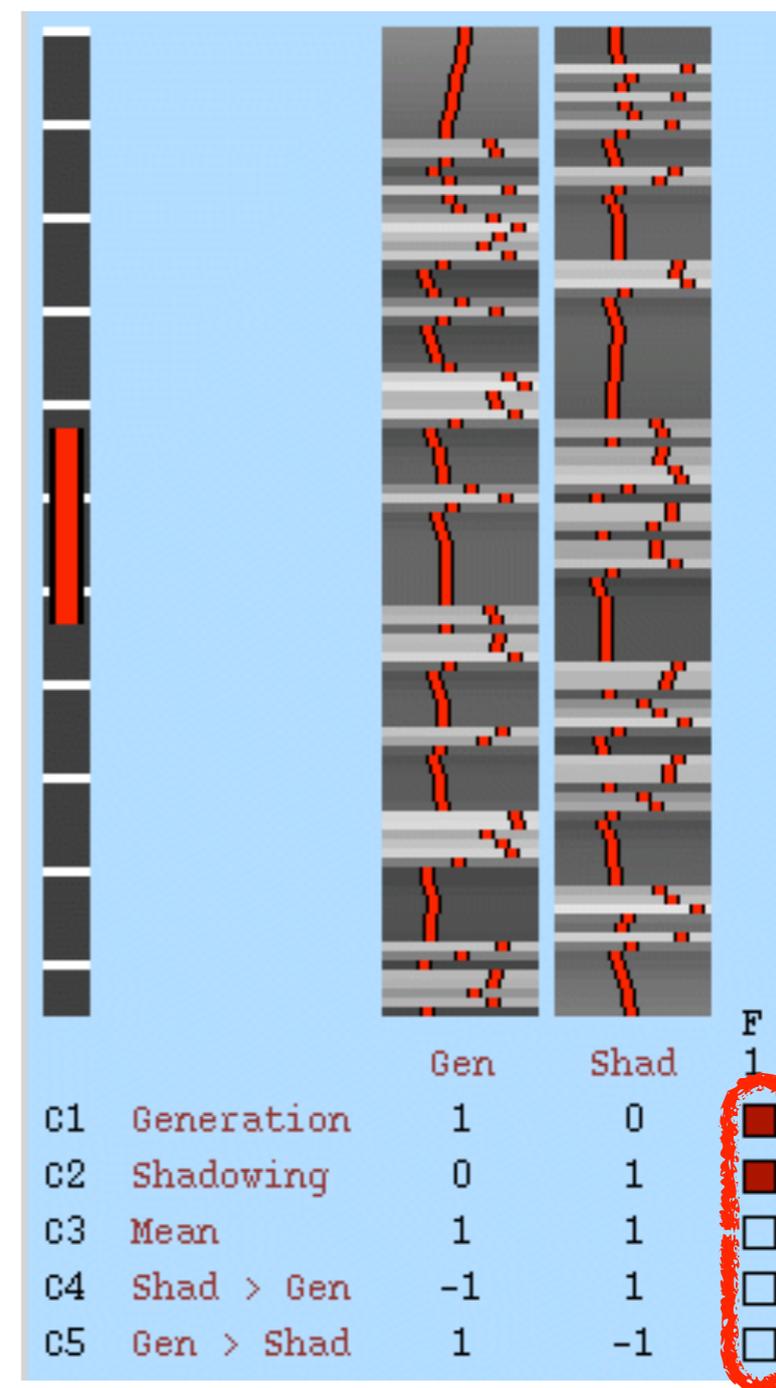


F-contrasts

- Two conditions: A, B
- Is any condition significant?

- Set of COPEs form an F-contrast
- Or: “Is there a significant amount of power in the data explained by the combination of the COPEs in the F-contrast?”

- F-contrast is F-distributed





Summary

- The GLM is used to summarise data in a few parameters that are pertinent to the experiment.
- GLM predicts how BOLD activity might change as a result of the experiment.
- We can test for significant effects by using t or f contrasts on the GLM parameters

That's all folks

