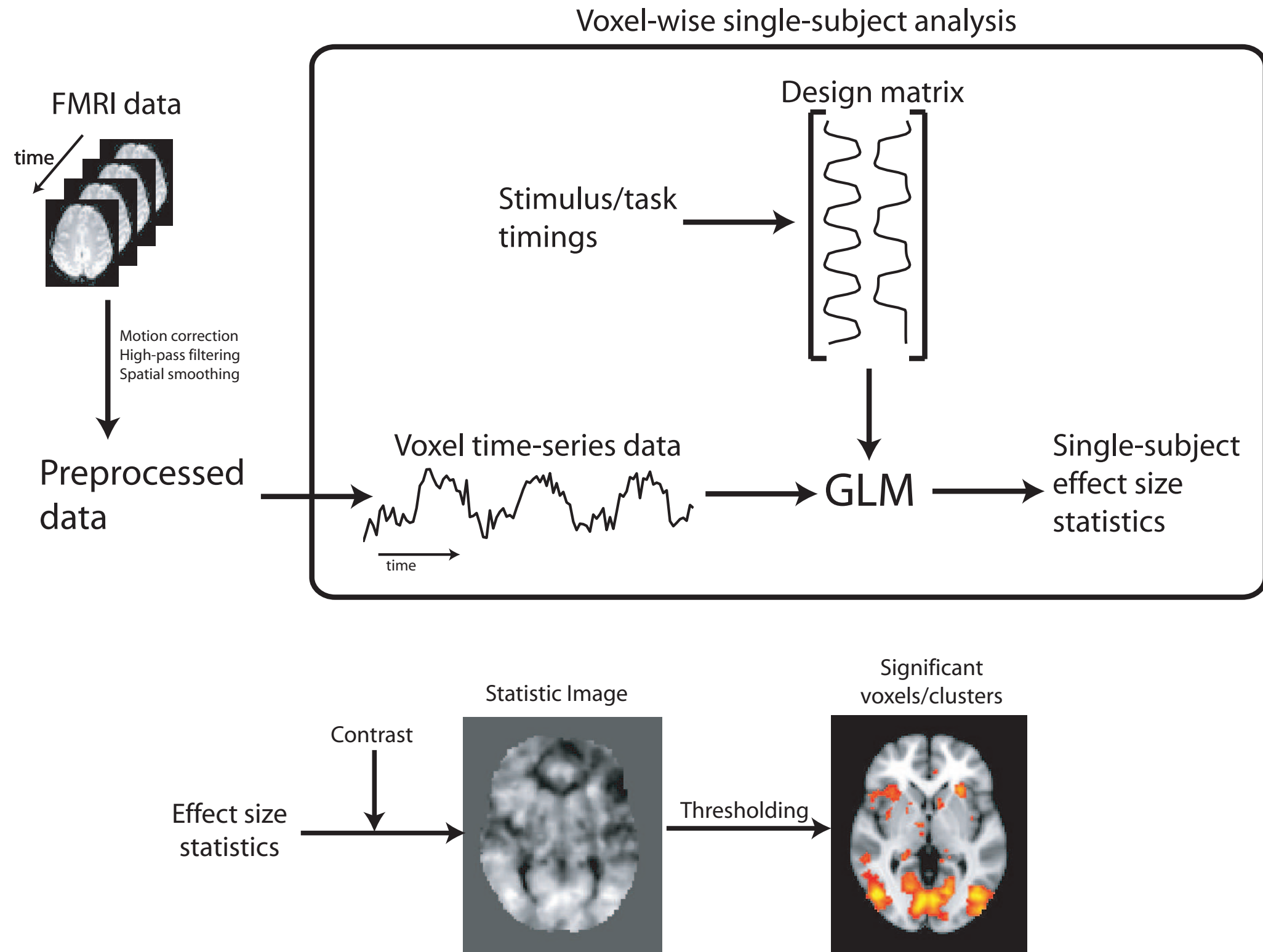




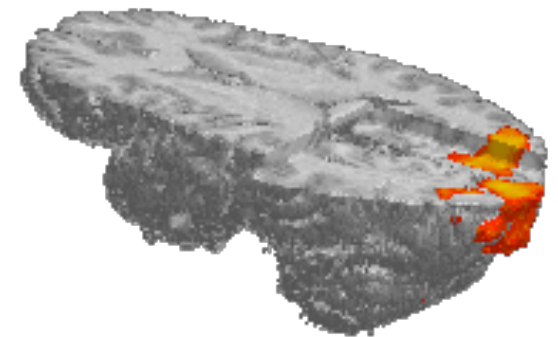
# Single-Session Analysis





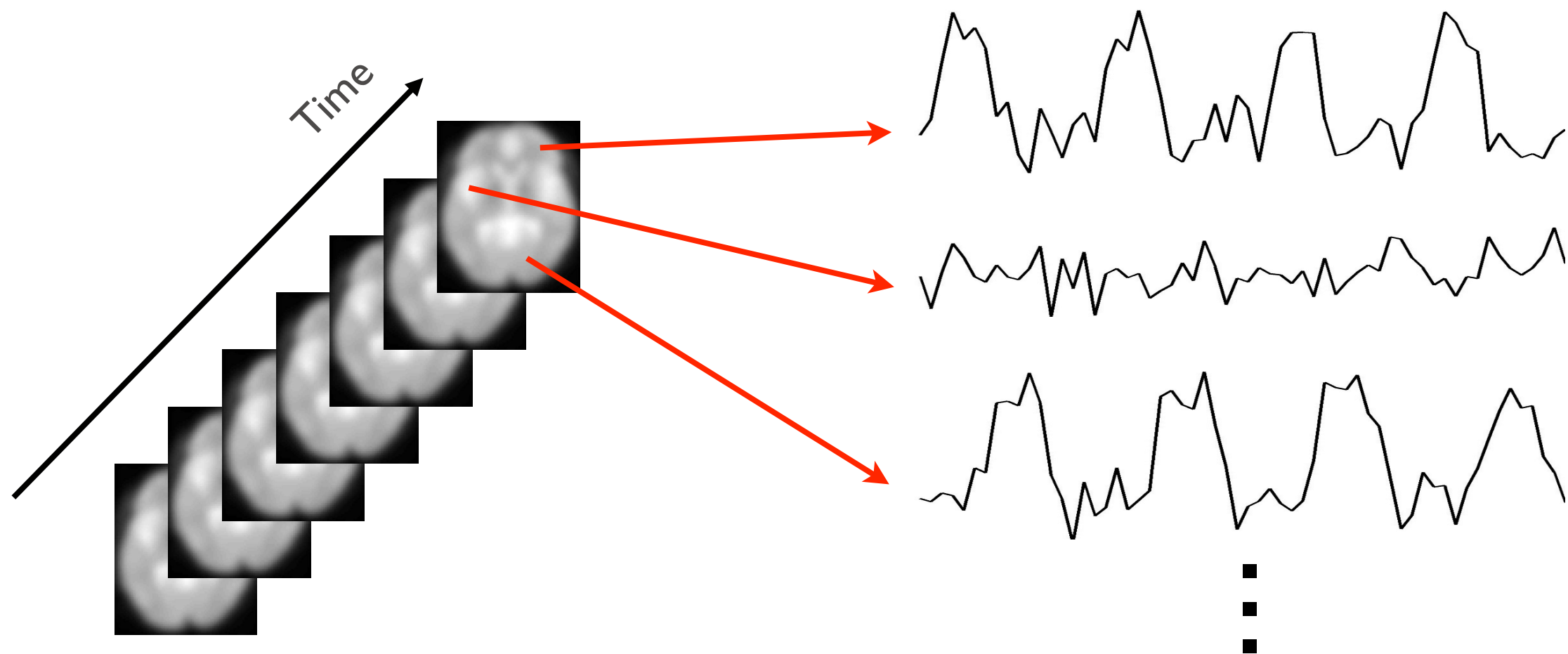
# FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





# Two different views of the data



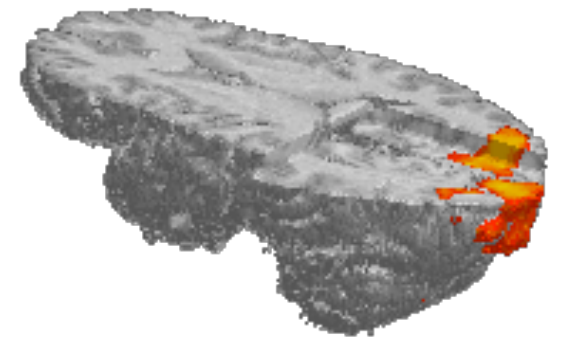
A “smallish”  
number of  
volumes

A **large**  
number of  
time series



# FMRI Modelling and Statistics

- **An example experiment**
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction

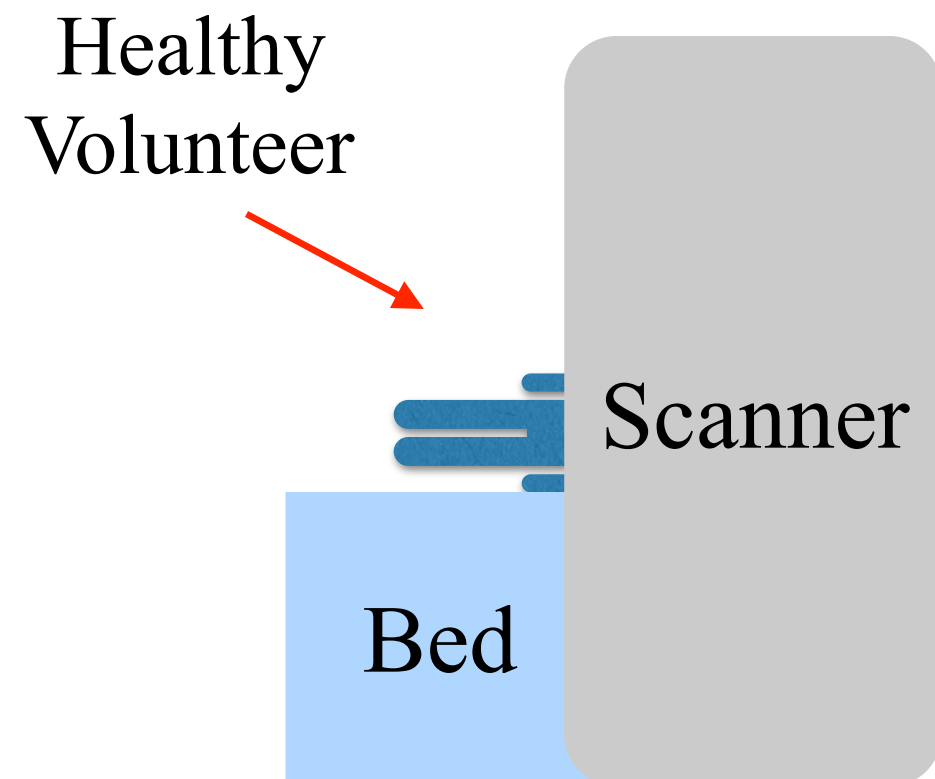
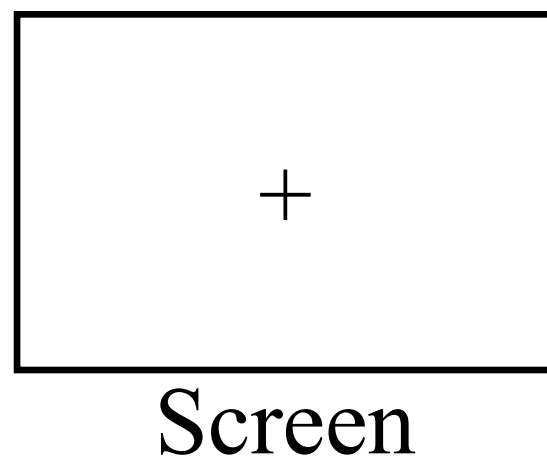




# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation



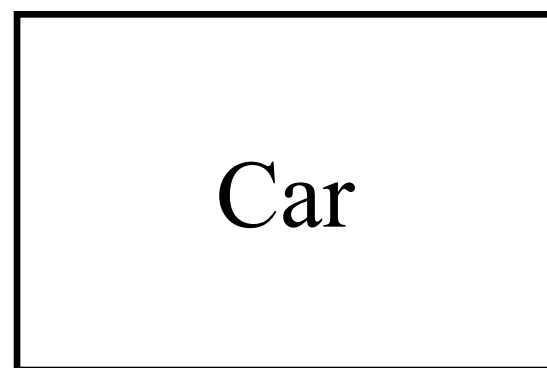


# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation

Noun is presented



Screen

Healthy  
Volunteer



Scanner

Bed

Verb is generated



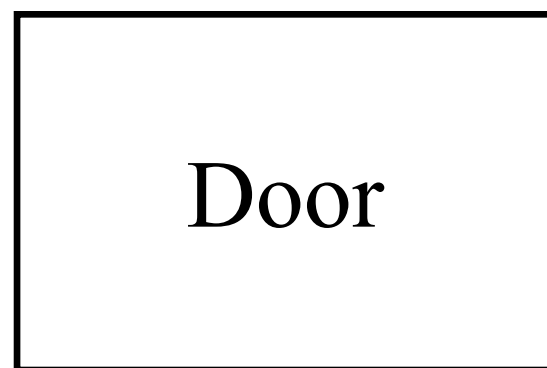


# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation

Noun is presented



Screen

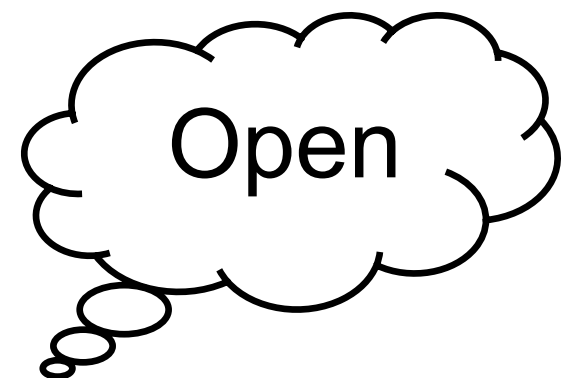
Healthy  
Volunteer



Scanner

Bed

Verb is generated



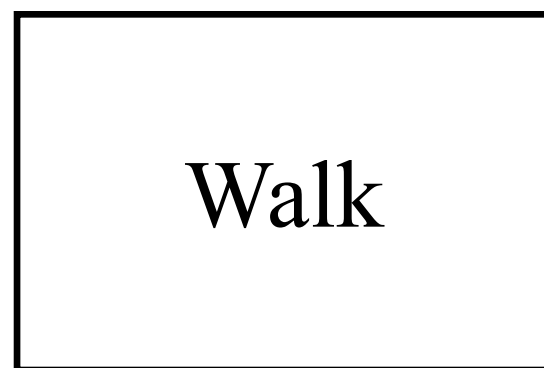


# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing

Verb is presented



Screen

Healthy  
Volunteer



Scanner

Bed

Verb is repeated





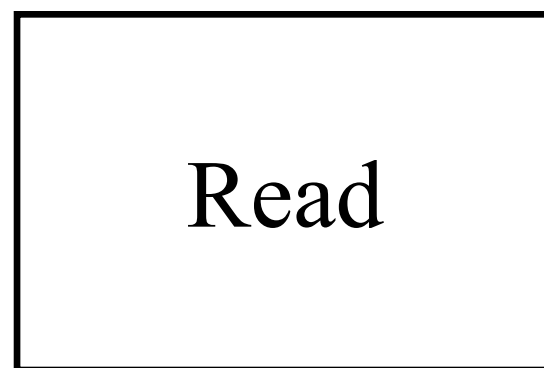


# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing

Verb is presented



Screen

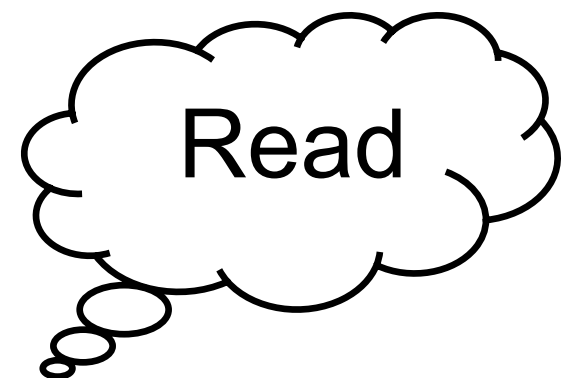
Healthy  
Volunteer



Scanner

Bed

Verb is repeated



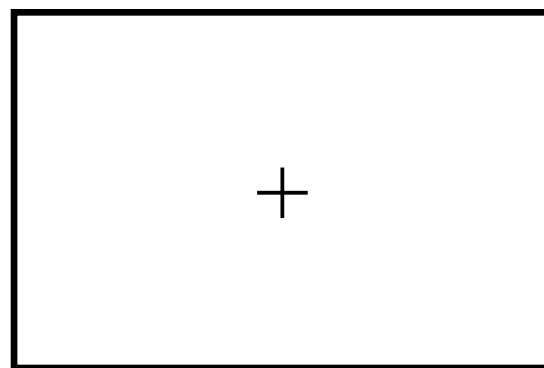


# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event

Crosshair is shown



Screen

Healthy  
Volunteer



Scanner

Bed



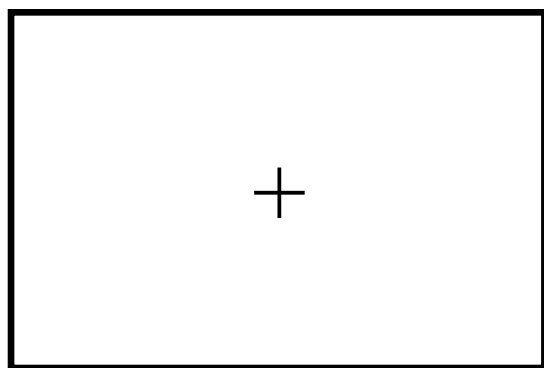


# An example experiment

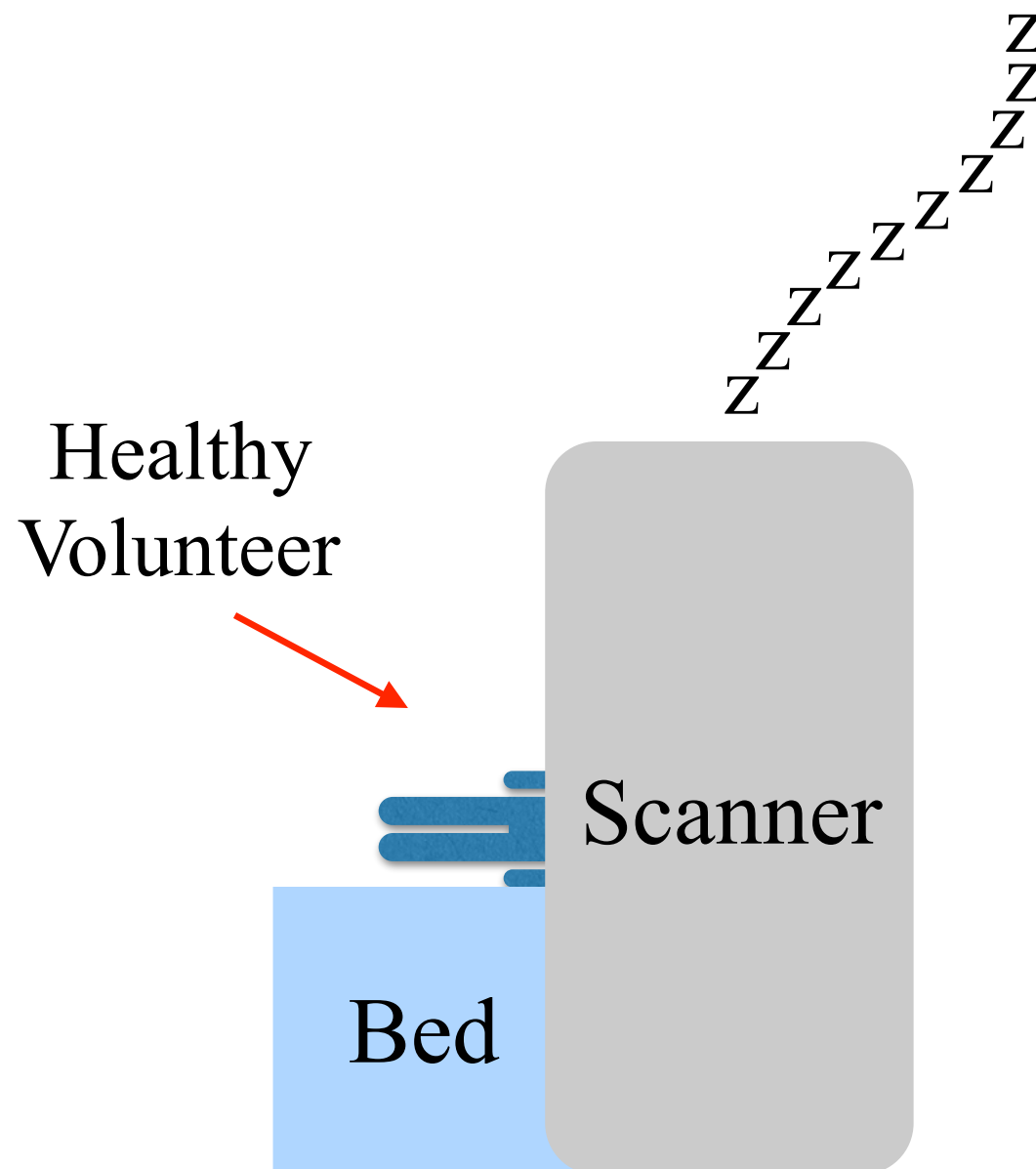
# An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event

# Crosshair is shown



# Screen



# Healthy Volunteer

# Scanner

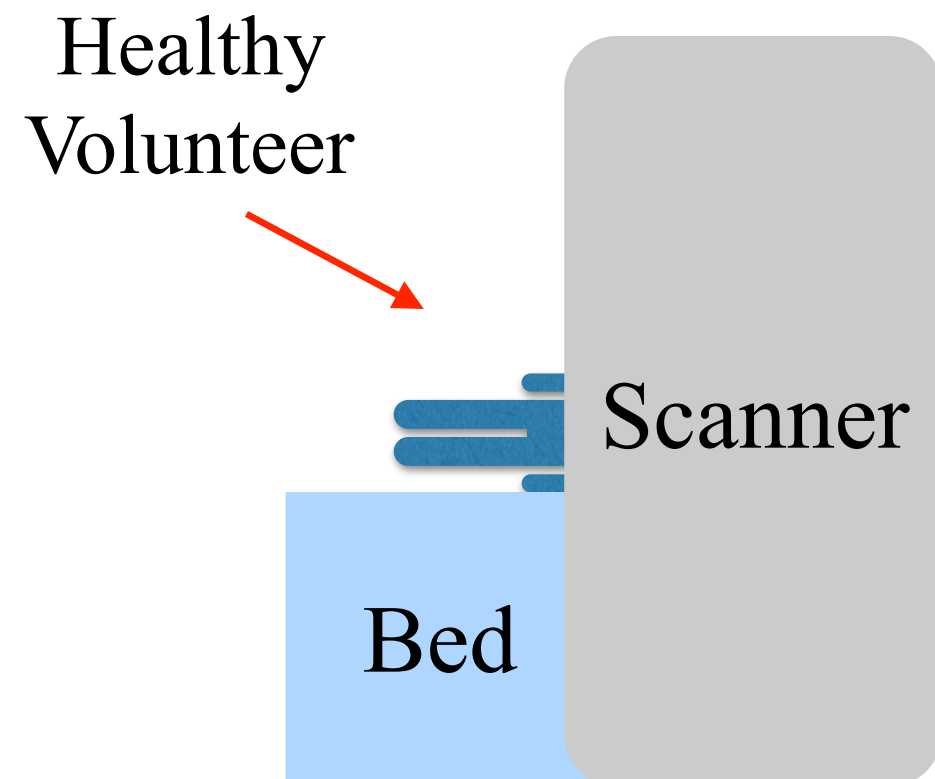
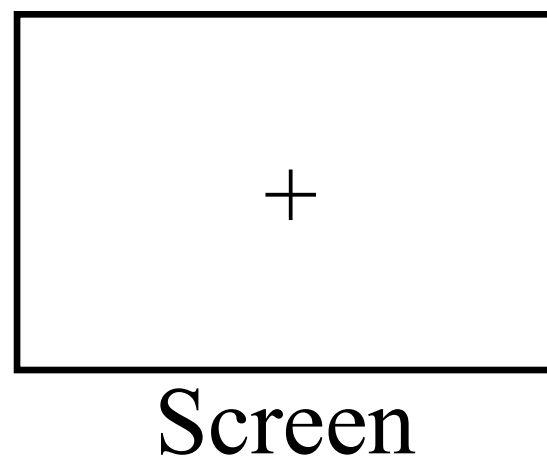
# Bed



# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event
- 6 sec ISI, random order

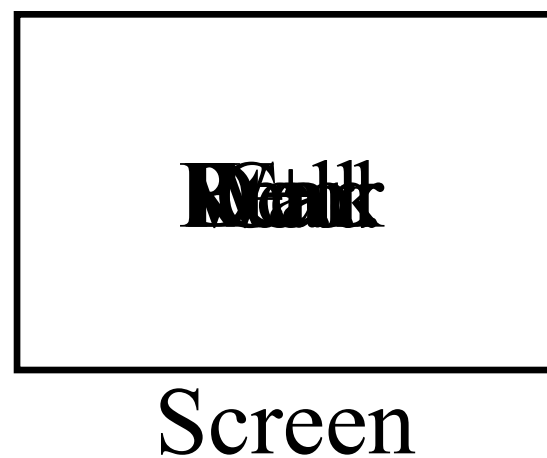




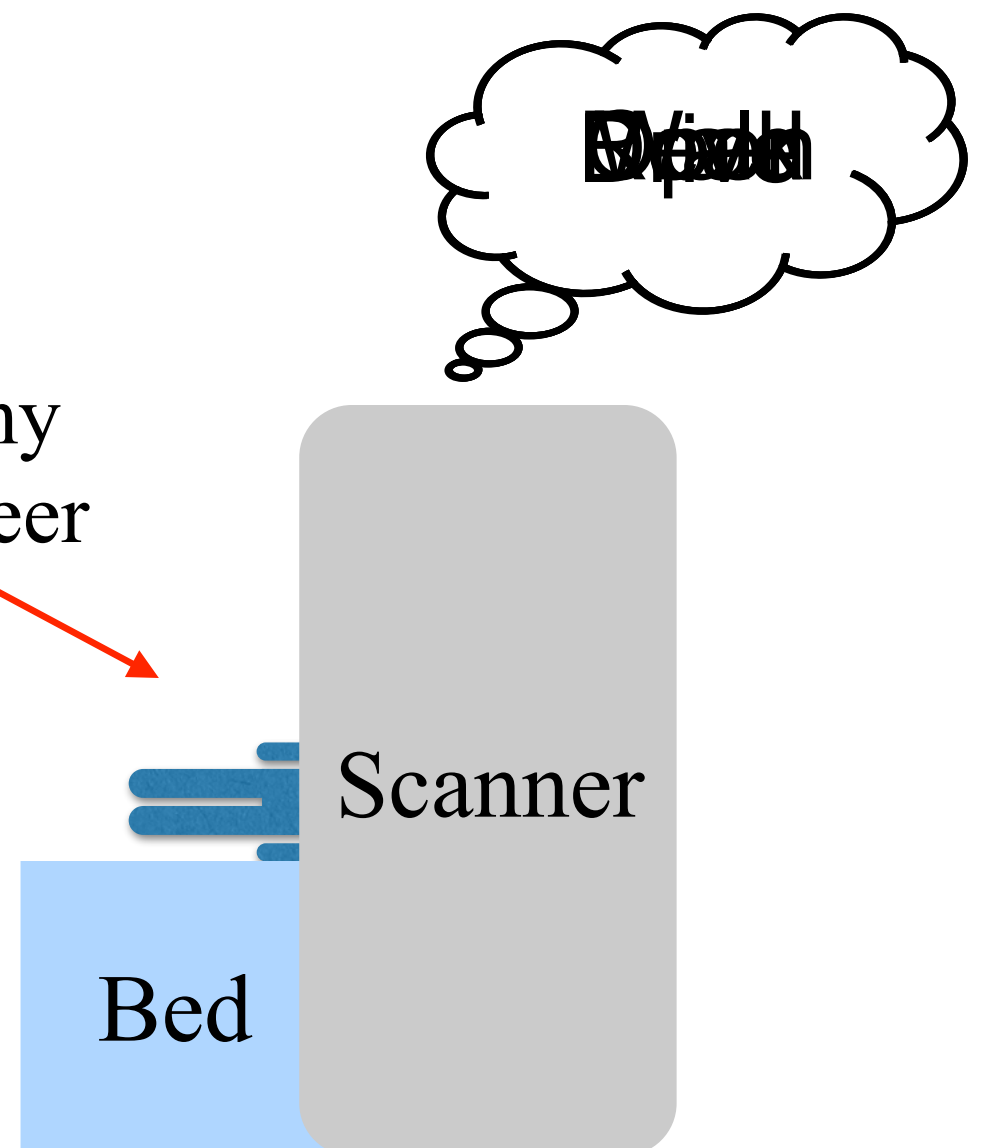
# An example experiment

An fMRI adaptation of a classical PET experiment

- Three types of events
- 1st type: Word Generation
- 2nd type: Word Shadowing
- 3rd type: Null event
- 6 sec ISI, random order
- For 24 events of each type



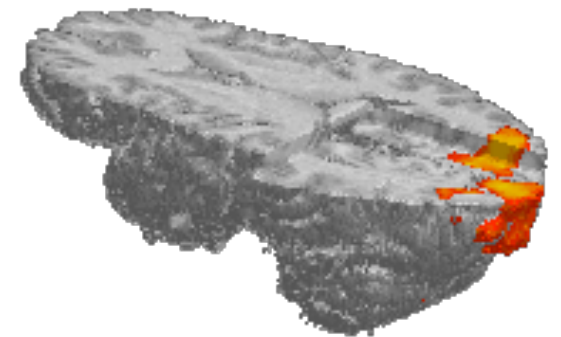
Healthy  
Volunteer





# FMRI Modelling and Statistics

- An example experiment
- **Multiple regression (GLM)**
- T and F Contrasts
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





# Building a model

Our task is now to build a model for that experiment

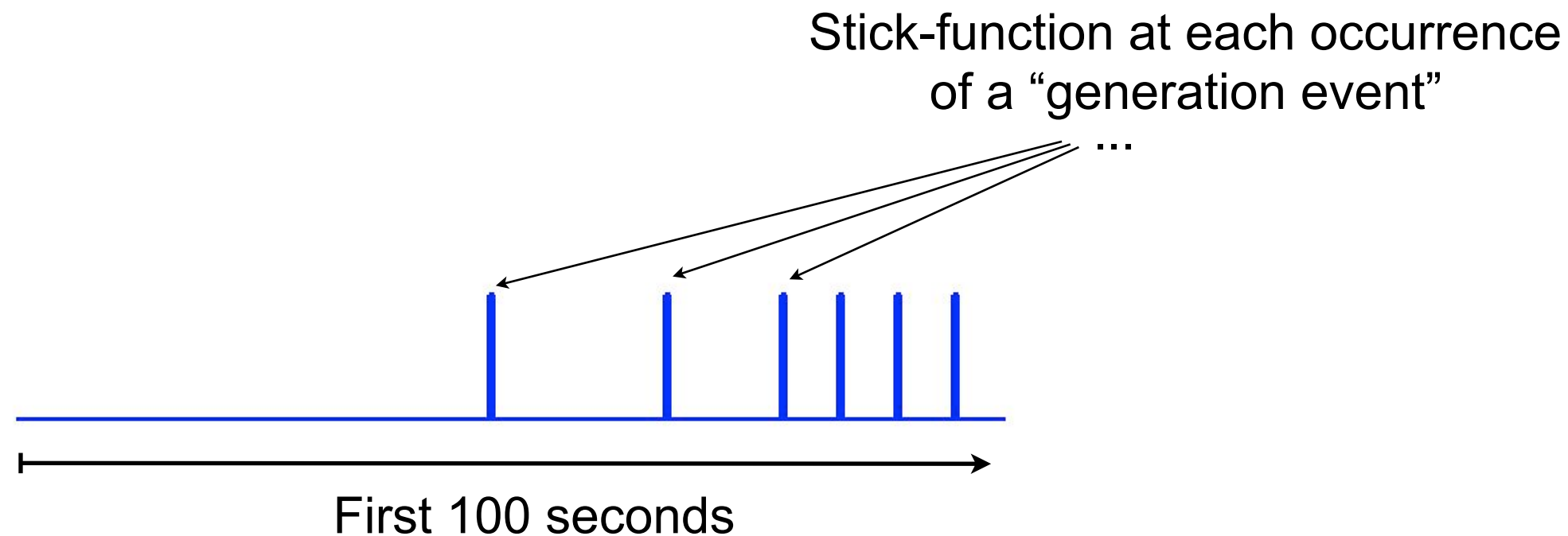
What is our predicted response to the word generation events?



# Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



Well, hardly like this...

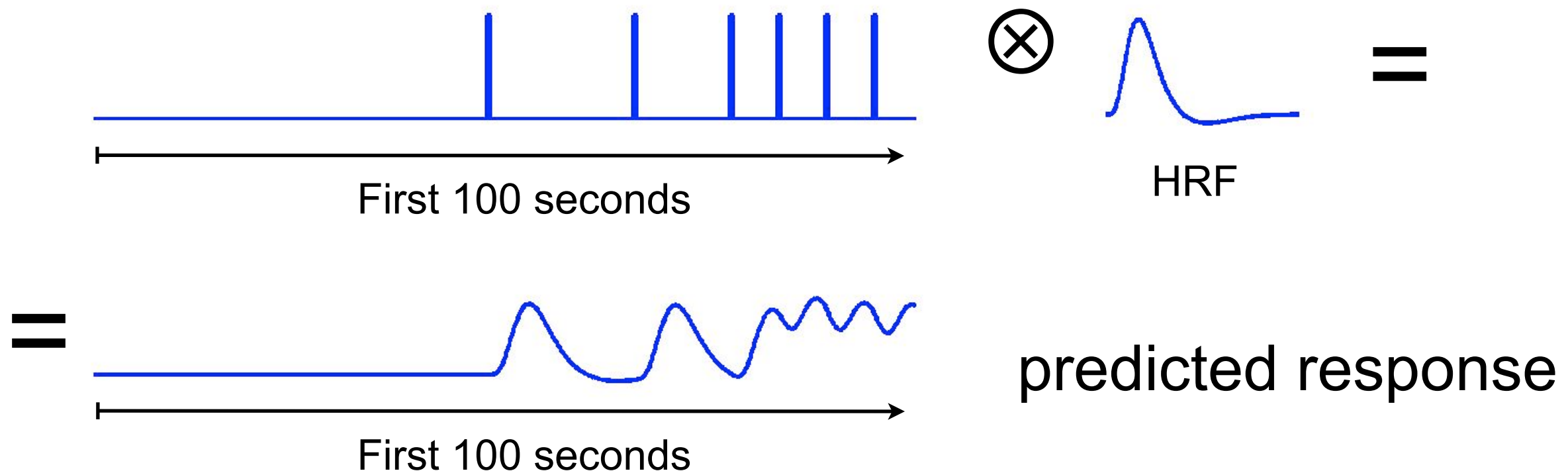




# Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



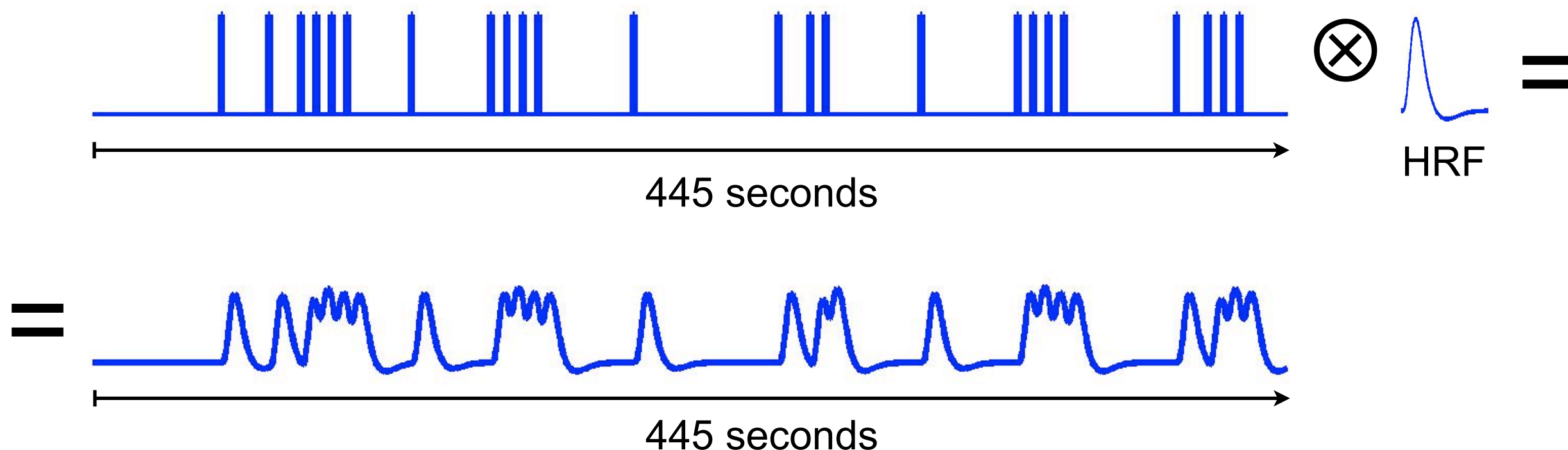
That looks better!



# Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



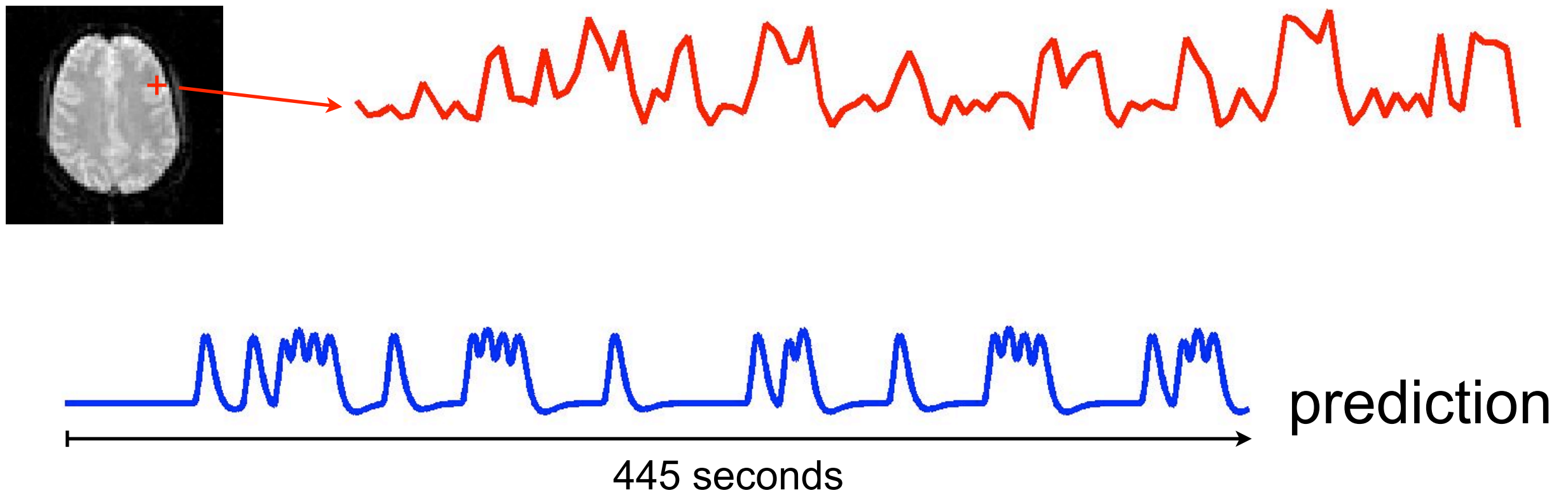
And this is the prediction for the whole time-series



# Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



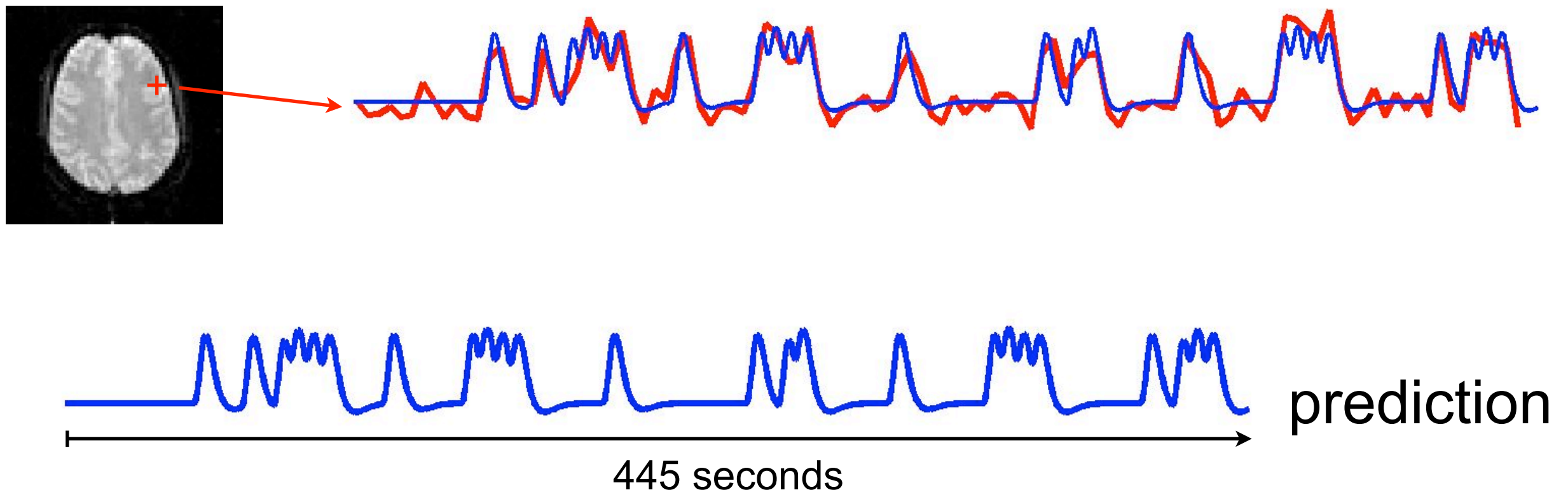
So, if we spot a time-series like this



# Building a model

Our task is now to build a model for that experiment

What is our predicted response to the word generation events?



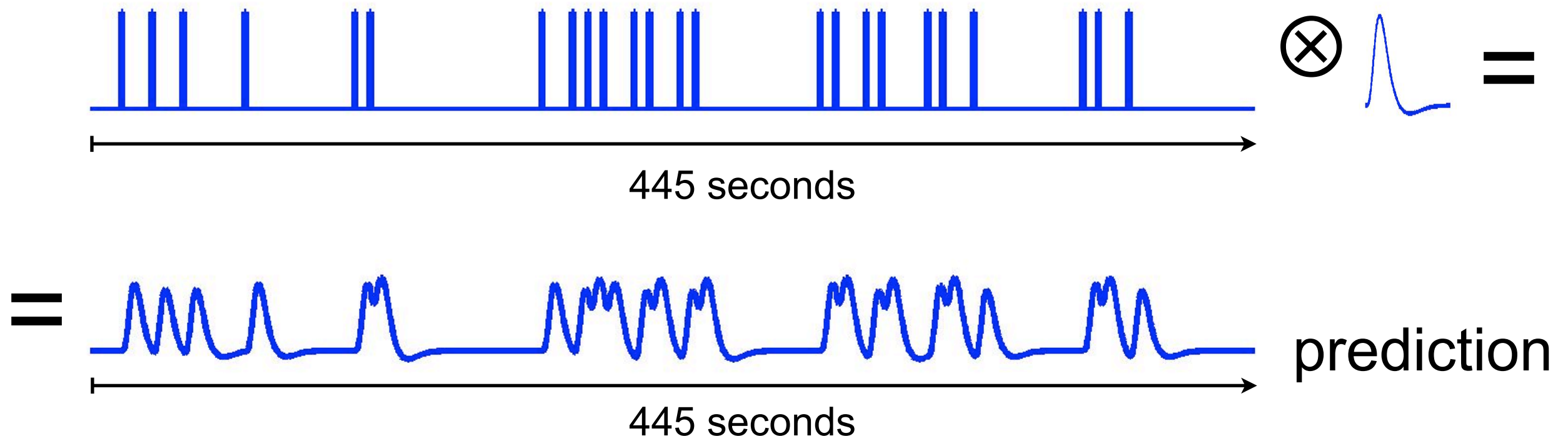
And then check it against our prediction  
we can conclude that this pixel is into word generation



# Building a model

Our task is now to build a model for that experiment

And we can do the same for the word shadowing events?



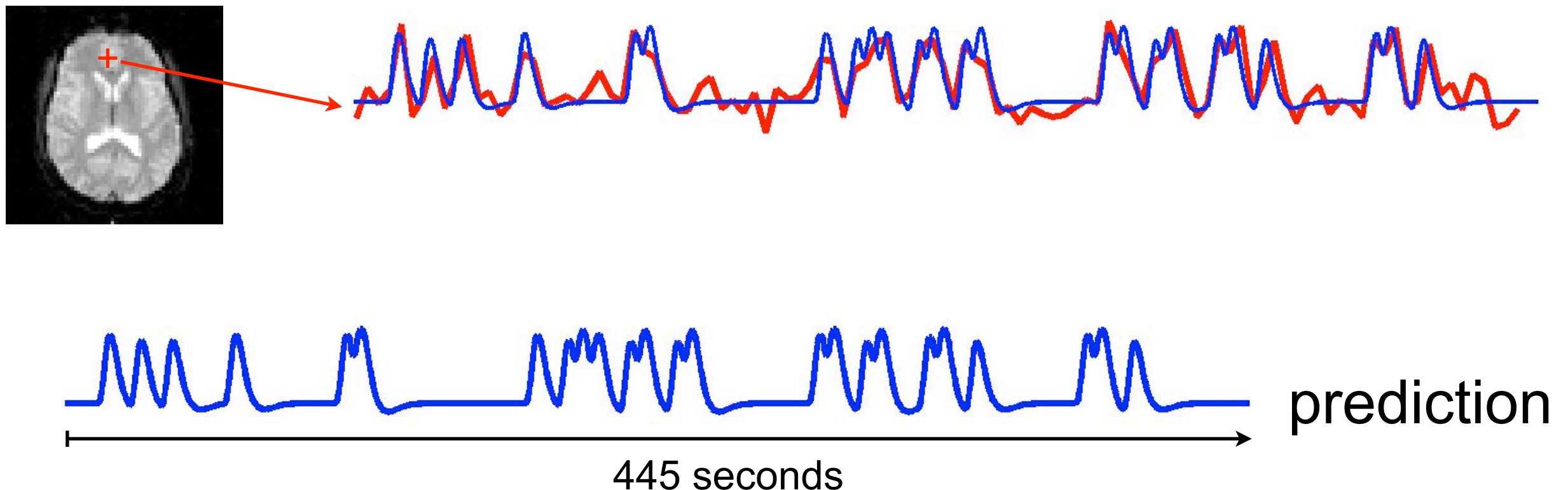
This time we used the onset times for the shadowing events to get the predicted brain response for those



# Building a model

Our task is now to build a model for that experiment

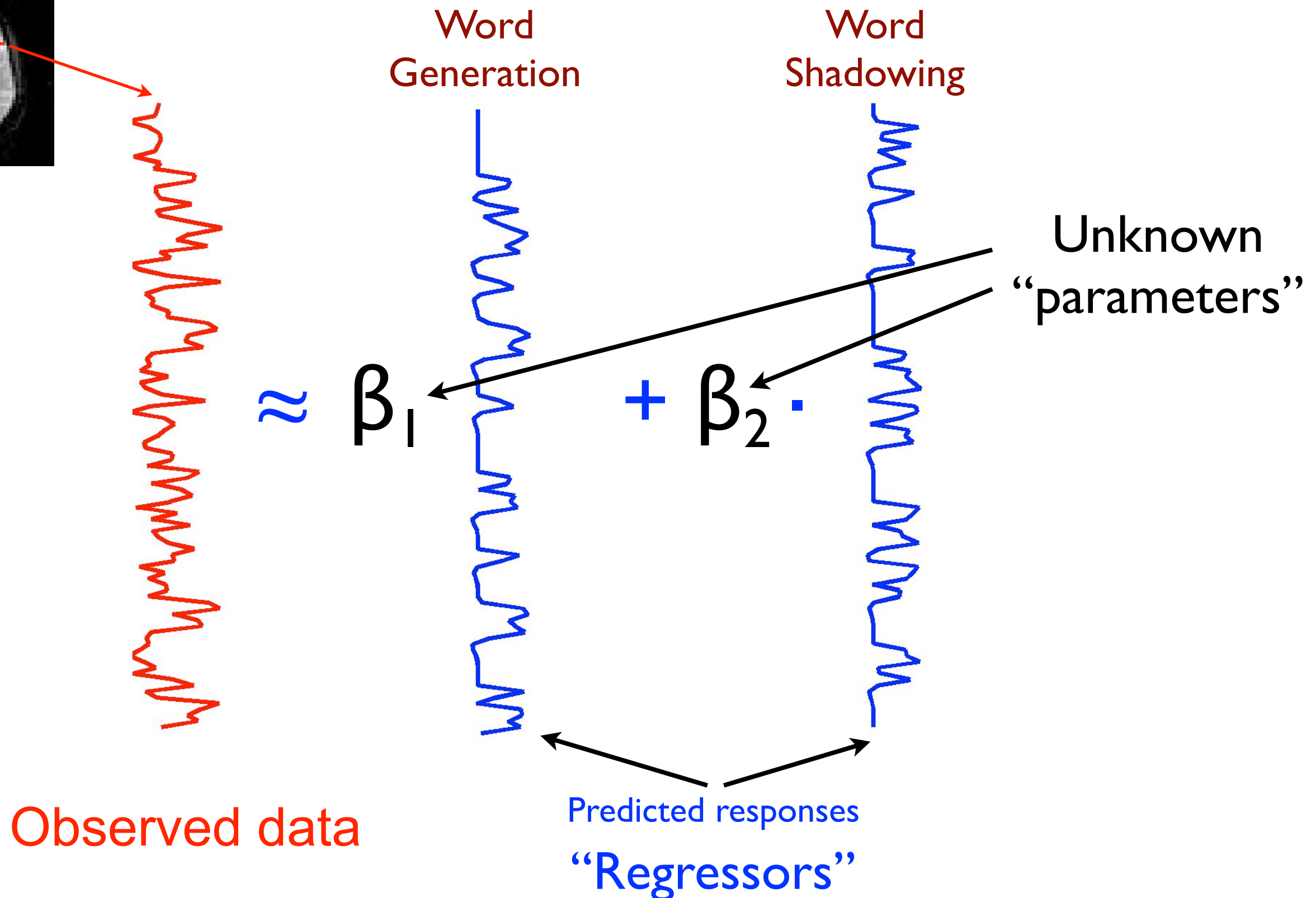
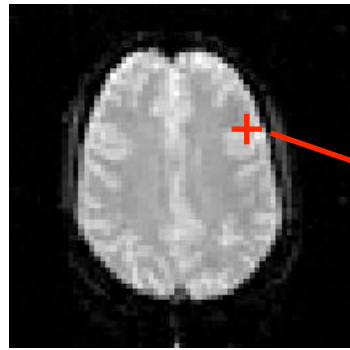
And we can do the same for the word shadowing events?



And we can look for voxels that match that

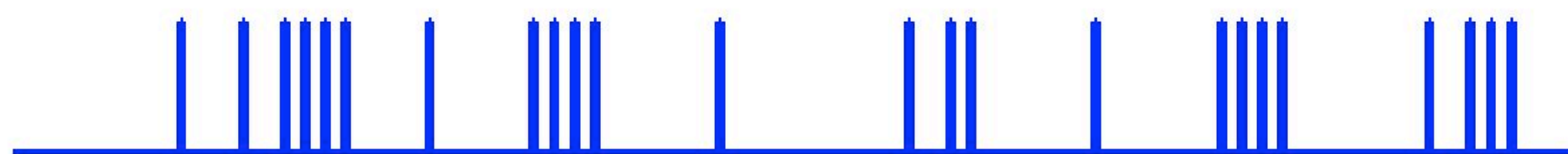


# Formalising it: Multiple regression

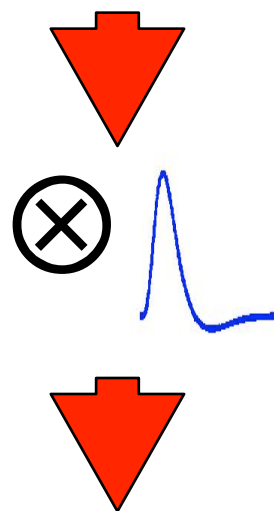




# Slight detour: Making regressors



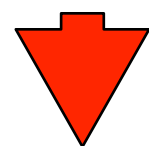
Event timings at  
“high” resolution



Convolve with  
HRF



Predictions at  
“high” resolution



Sub-sample at  $T_R$   
of experiment



Regressor

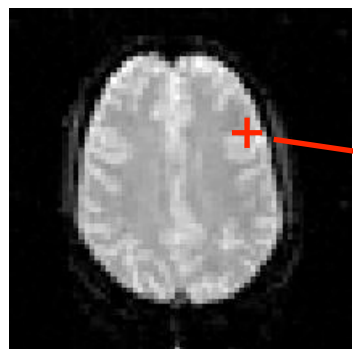




# Estimation:

## Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



Word  
Generation

Word  
Shadowing



$\approx$

$\beta_1$   
0.5



$+$   $\beta_2$   $\cdot$   
0.5



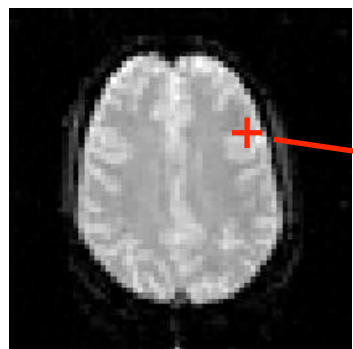
Let's try  
these  
parameter  
values



# Estimation:

## Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



Word  
Generation

Word  
Shadowing

$$\approx \beta_1 \cdot 0.5 + \beta_2 \cdot 0.5$$

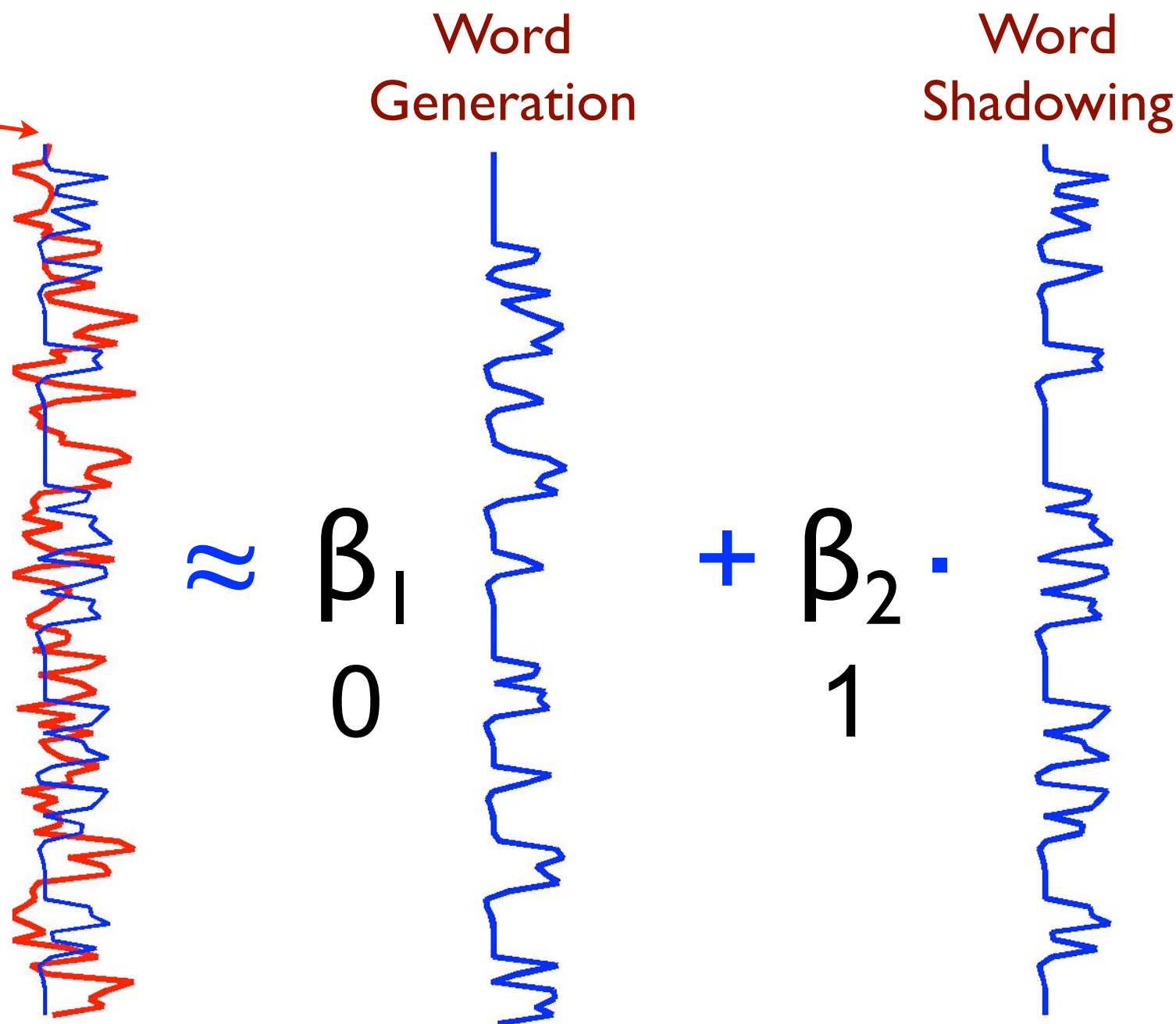
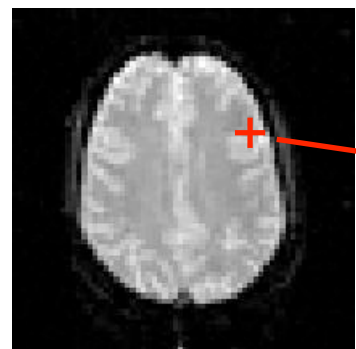
Hmm, not a  
great fit



# Estimation:

## Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



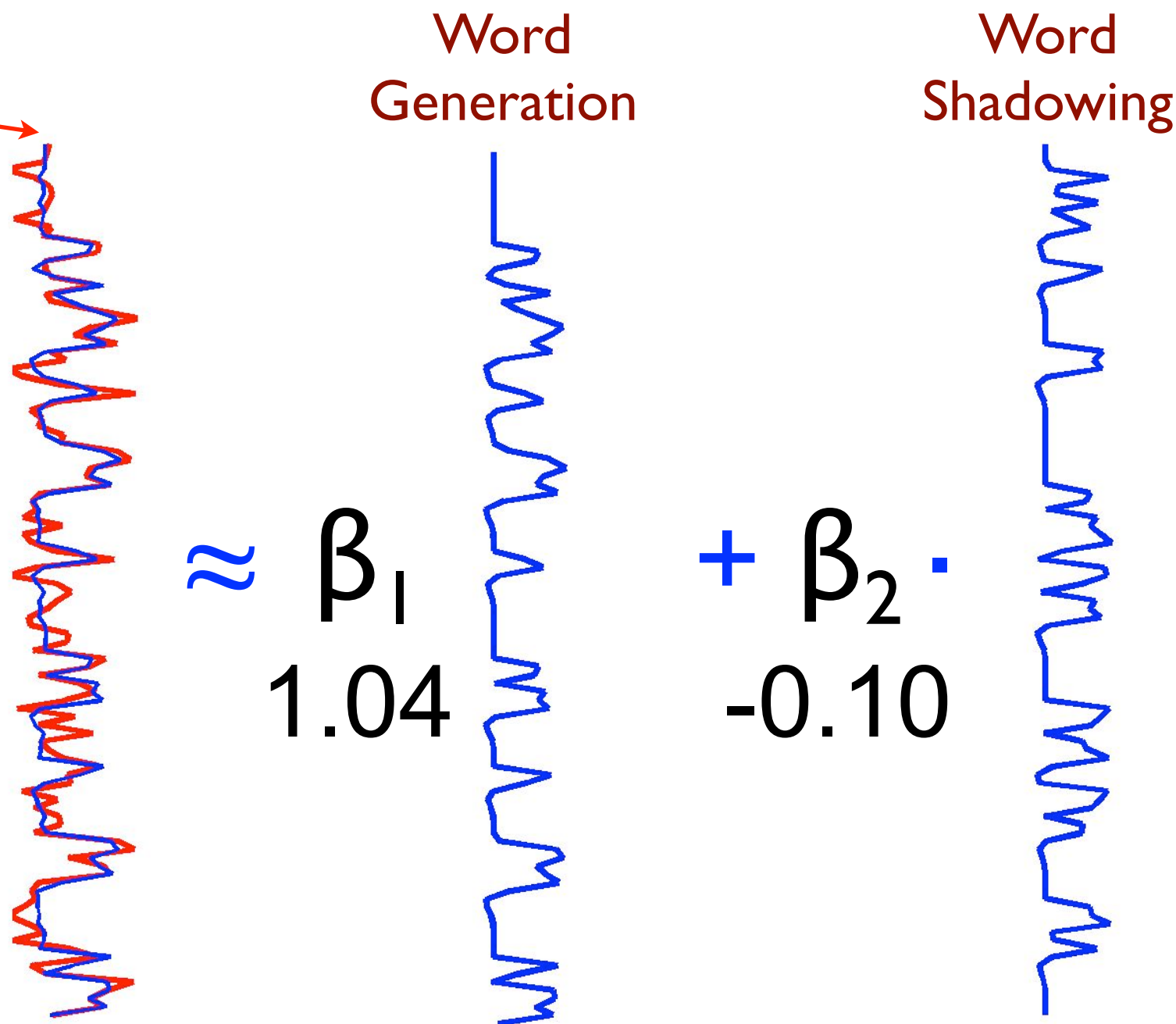
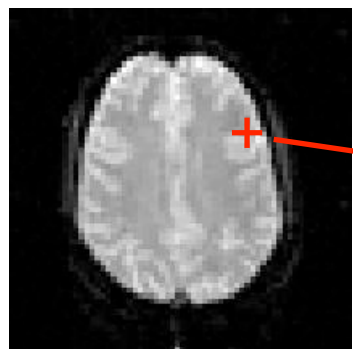
Oh dear,  
even worse



# Estimation:

## Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data.



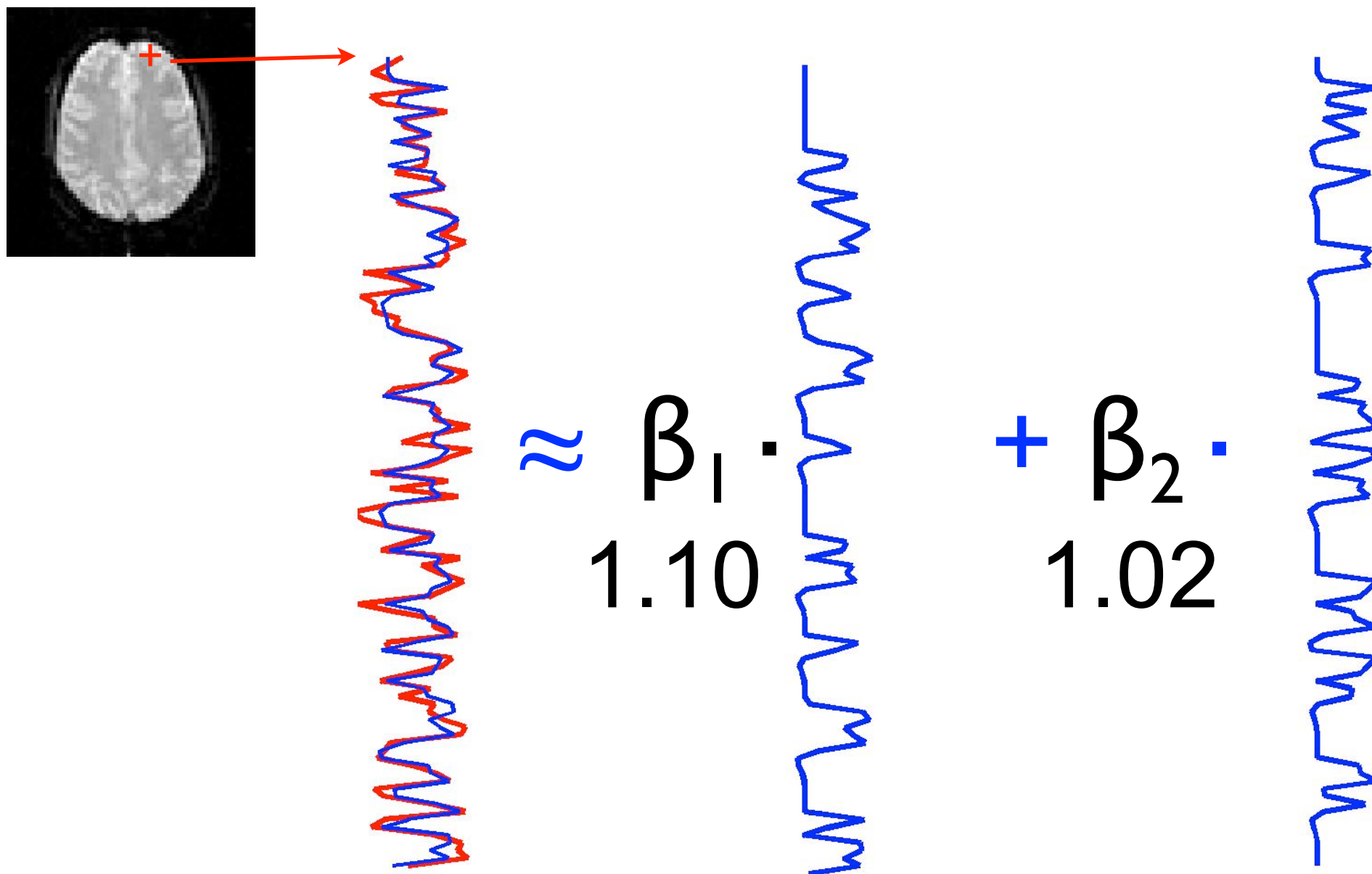
But that looks good



# Estimation:

## Finding the “best” parameter values

- The estimation entails finding the parameter values such that the linear combination “best” fits the data



And different voxels yield different parameters

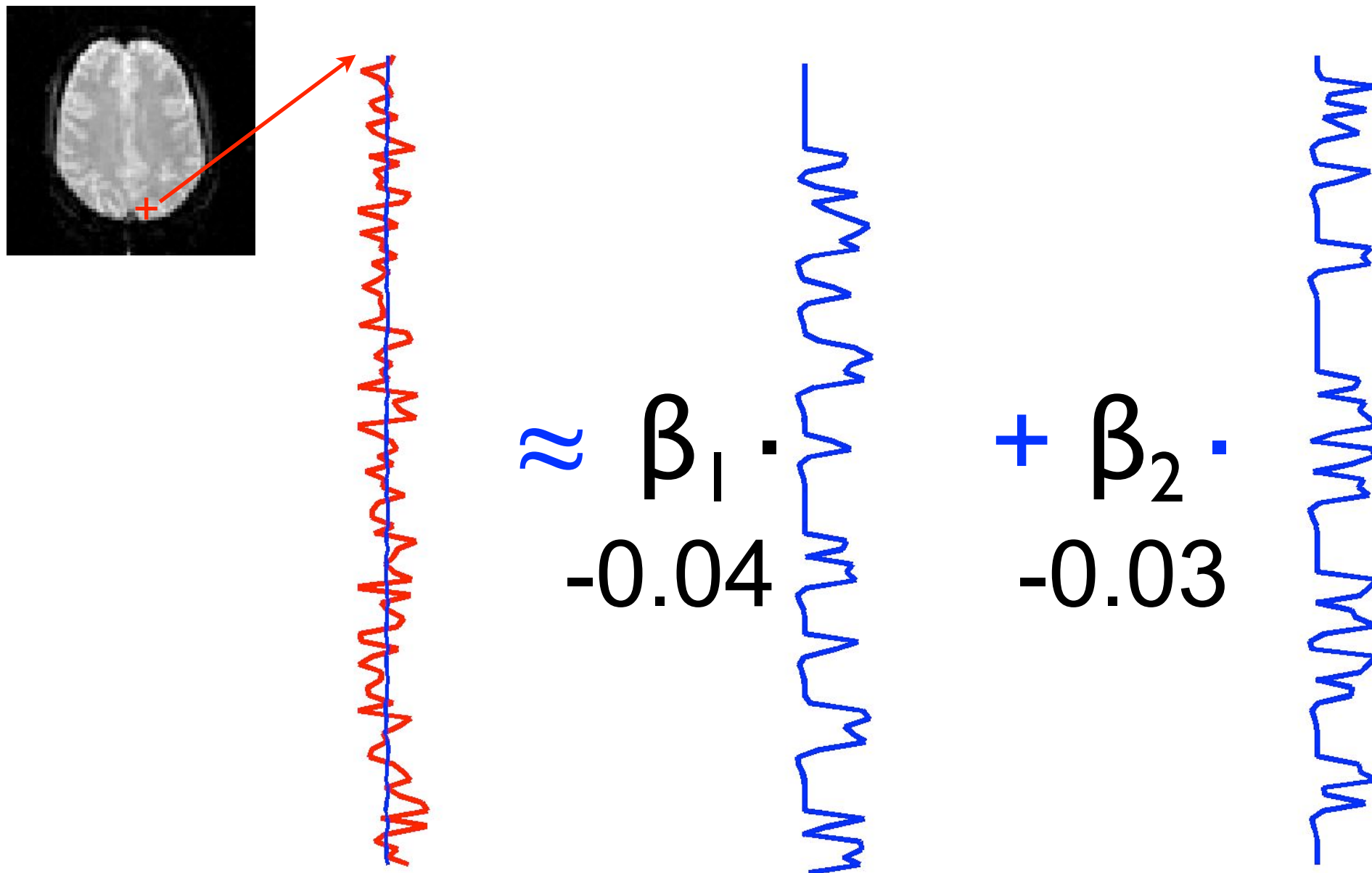




# Estimation:

## Finding the “best” parameter values

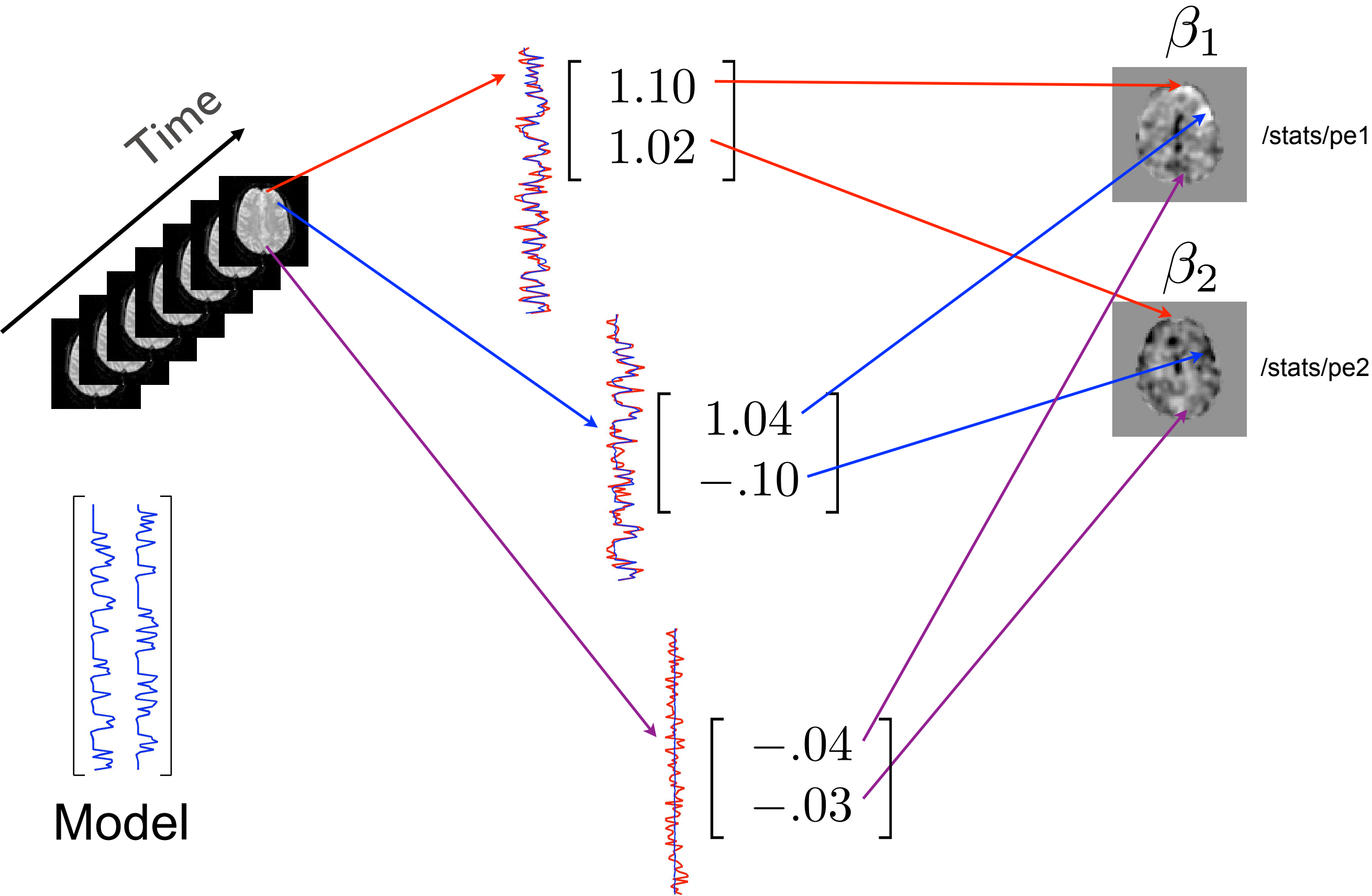
- The estimation entails finding the parameter values such that the linear combination “best” fits the data



And different voxels yield different parameters



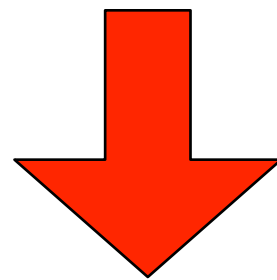
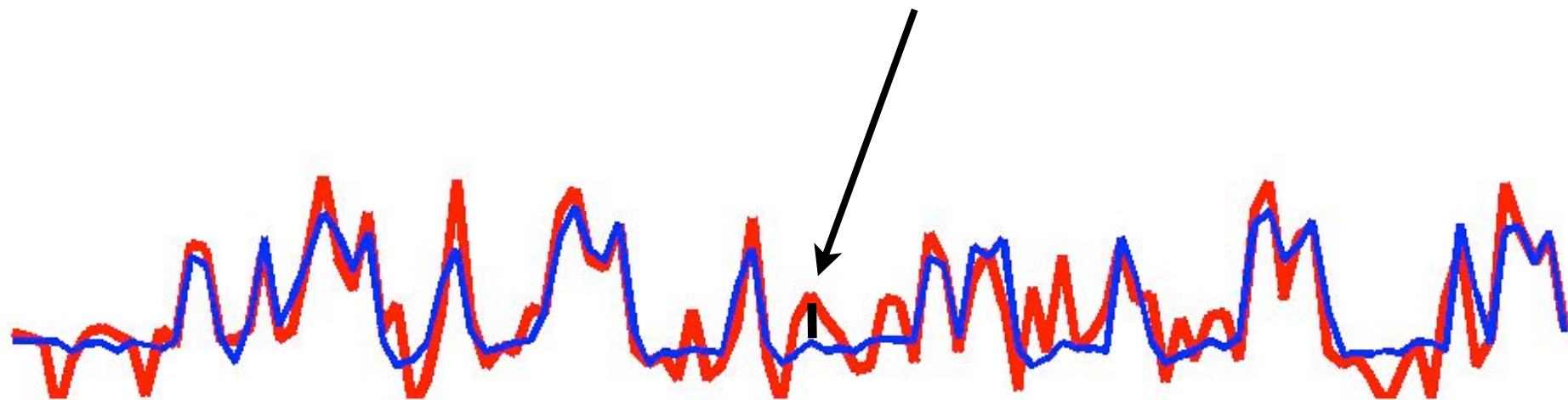
# One model to fit them all





And we can also estimate the residual error

Difference between data and best fit: "Residual error"

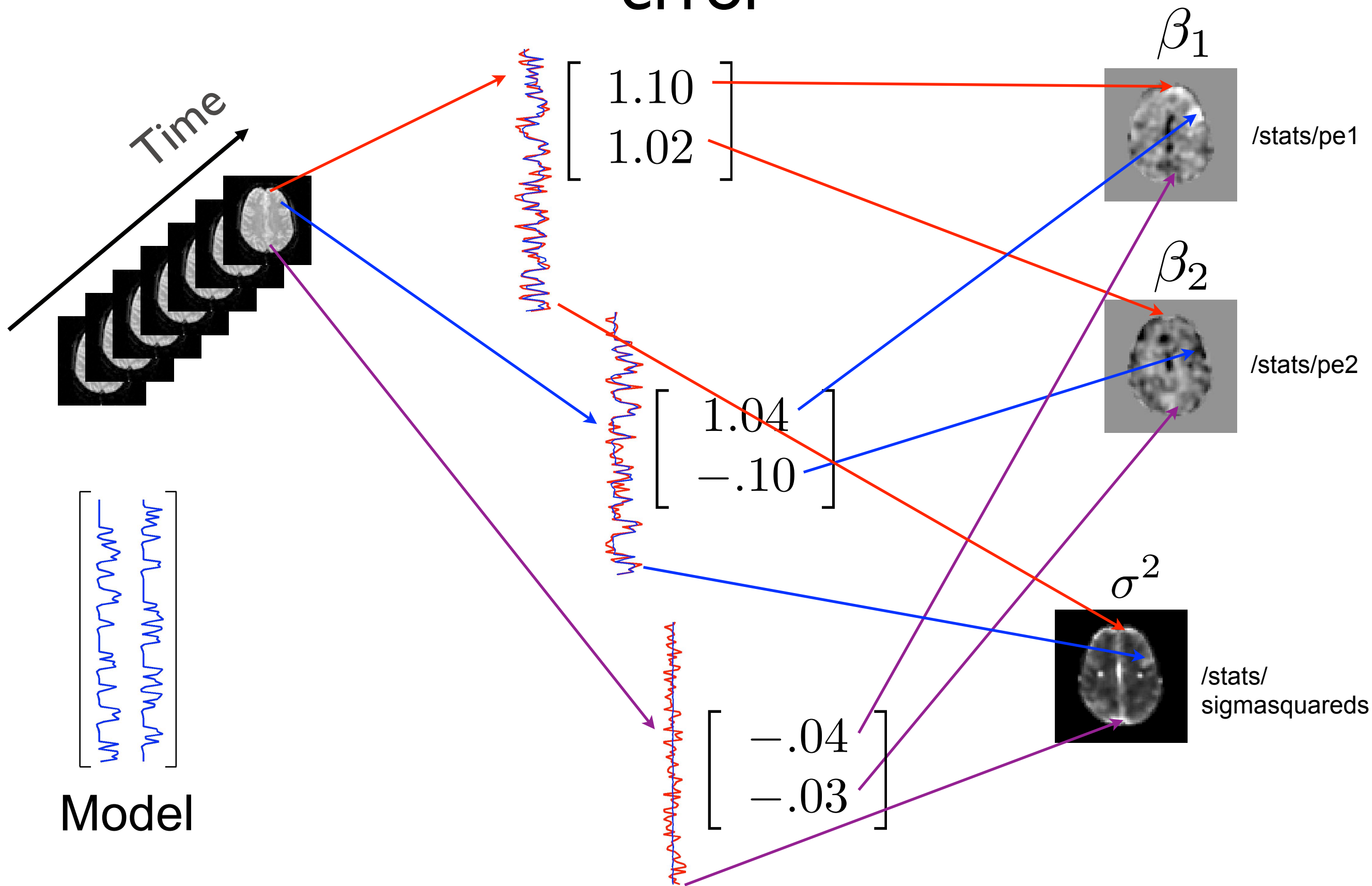


Residual errors





# And we can also estimate the residual error





# Summary of what we learned so far

- The “Model” consists of a set of “regressors” i.e. tentative time series that we expect to see as a response to our stimulus
- The model typically consists of our stimulus functions convolved by the HRF
- The estimation entails finding the parameter values such that the linear combination of regressors “best” fits the data
- Every voxel has its own unique parameter values, that is how a single model can fit so many different time series
- We can also get an estimate of the error through the “residuals”



# General Linear Model (GLM)

- This is placed into the General Linear Model (GLM) framework

Regressor, Explanatory Variable (EV)

Regression parameters, Effect sizes

Design Matrix

Gaussian noise (temporal autocorrelation)

Data from a voxel

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{e}$$

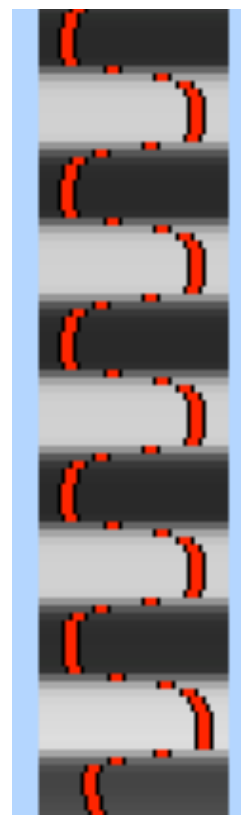
The diagram illustrates the General Linear Model (GLM) framework. It shows the equation  $\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{e}$  with various components labeled.  $\mathbf{y}$  is labeled 'Data from a voxel' and is represented by a red waveform.  $\mathbf{X}$  is labeled 'Design Matrix' and is represented by a matrix containing two columns,  $\mathbf{x}_1$  and  $\mathbf{x}_2$ , which are labeled 'Regressor, Explanatory Variable (EV)' and are represented by blue waveforms.  $\boldsymbol{\beta}$  is labeled 'Regression parameters, Effect sizes' and is represented by a vector containing  $\beta_1$  and  $\beta_2$ .  $\mathbf{e}$  is labeled 'Gaussian noise (temporal autocorrelation)' and is represented by a black waveform. The equation is shown as  $\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{e}$ .



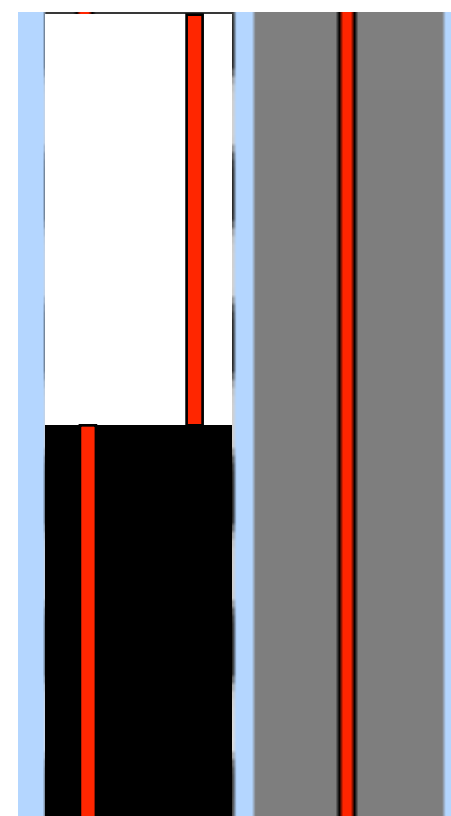
# “Demeaning” and the GLM

- The mean value is uninteresting in an FMRI session
- There are two equivalent options:
  - 1.remove the mean from the data and don't model it
  - 2.put a term into the model to account for the mean

option #1



option #2



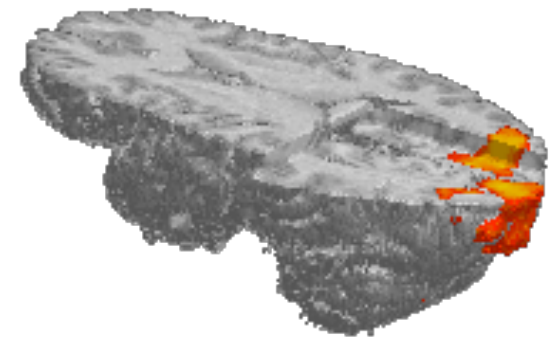
In FSL we use option #1 for first-level analyses and #2 for higher-level analyses

A consequence is that the baseline condition in first-level analysis is **NOT** explicitly modelled (in FSL)



# FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- **T and F Contrasts**
- Null hypothesis testing
- The residuals
- Thresholding: multiple comparison correction





# t-contrasts

- A contrast of parameter estimates (COPE) is a linear combination of PEs:

$$[1 \ 0]: \text{COPE} = 1 \times \hat{\beta}_1 + 0 \times \hat{\beta}_2 = \hat{\beta}_1$$

$$[1 \ -1]: \text{COPE} = 1 \times \hat{\beta}_1 + -1 \times \hat{\beta}_2 = \hat{\beta}_1 - \hat{\beta}_2$$

- Test null hypothesis that COPE=0

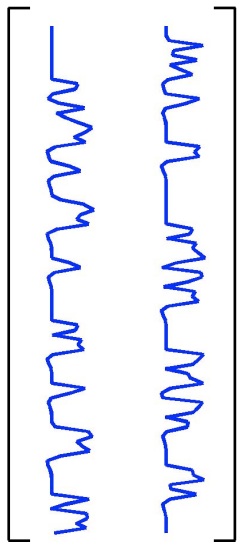
**t-statistic:**  $t = \frac{\text{COPE}}{\text{std}(\text{COPE})}$



# t-contrasts

$$t = \frac{COPE}{std(COPE)}$$

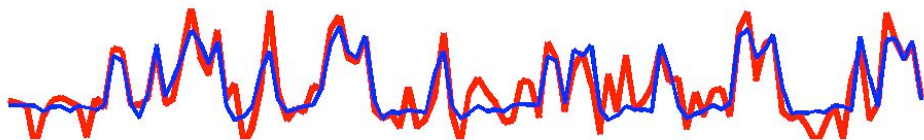
Depends on



The Model

$[1 \ 0]$

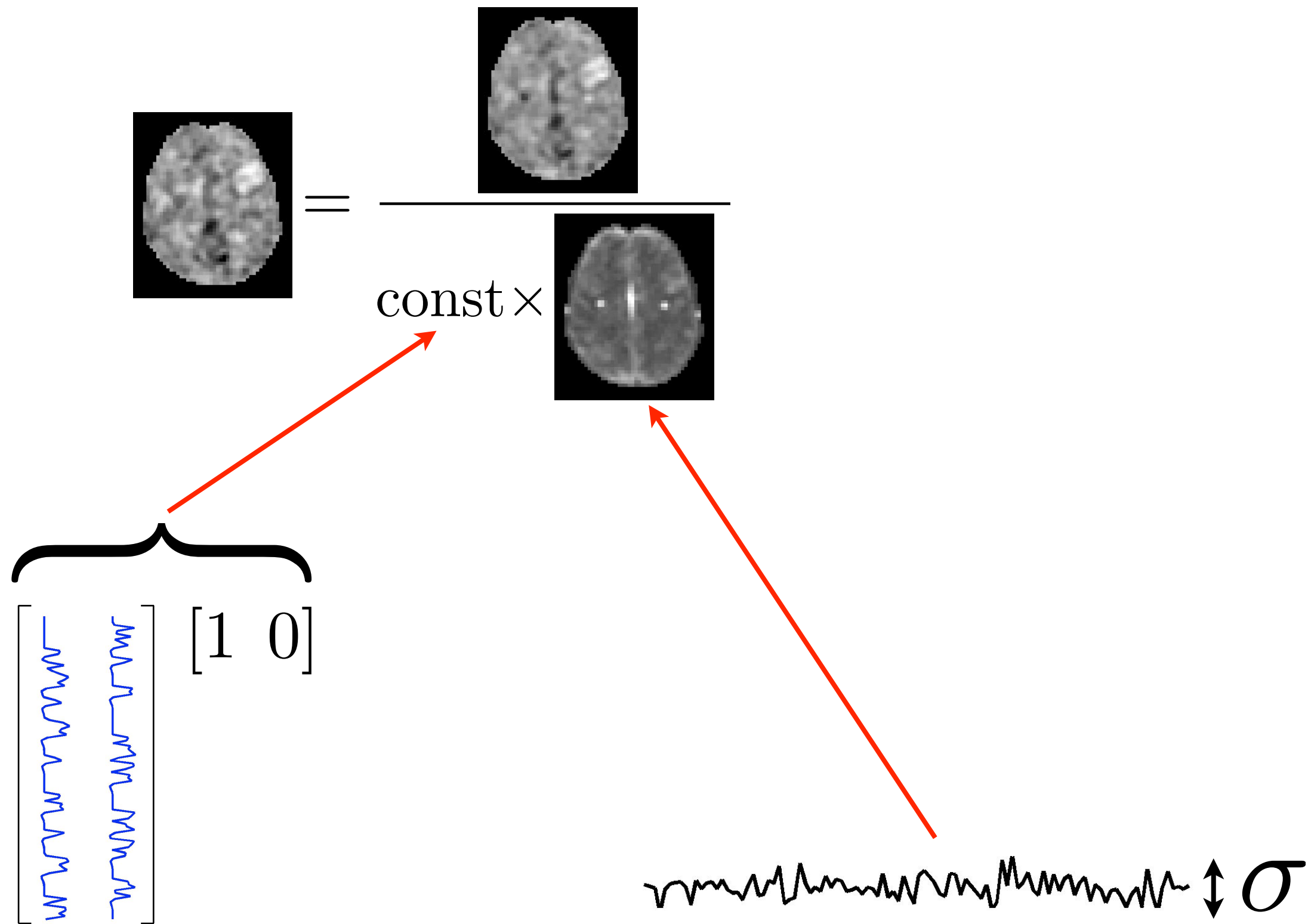
,the Contrast



and the Residual Error



# t-contrasts



The Model & the Contrast

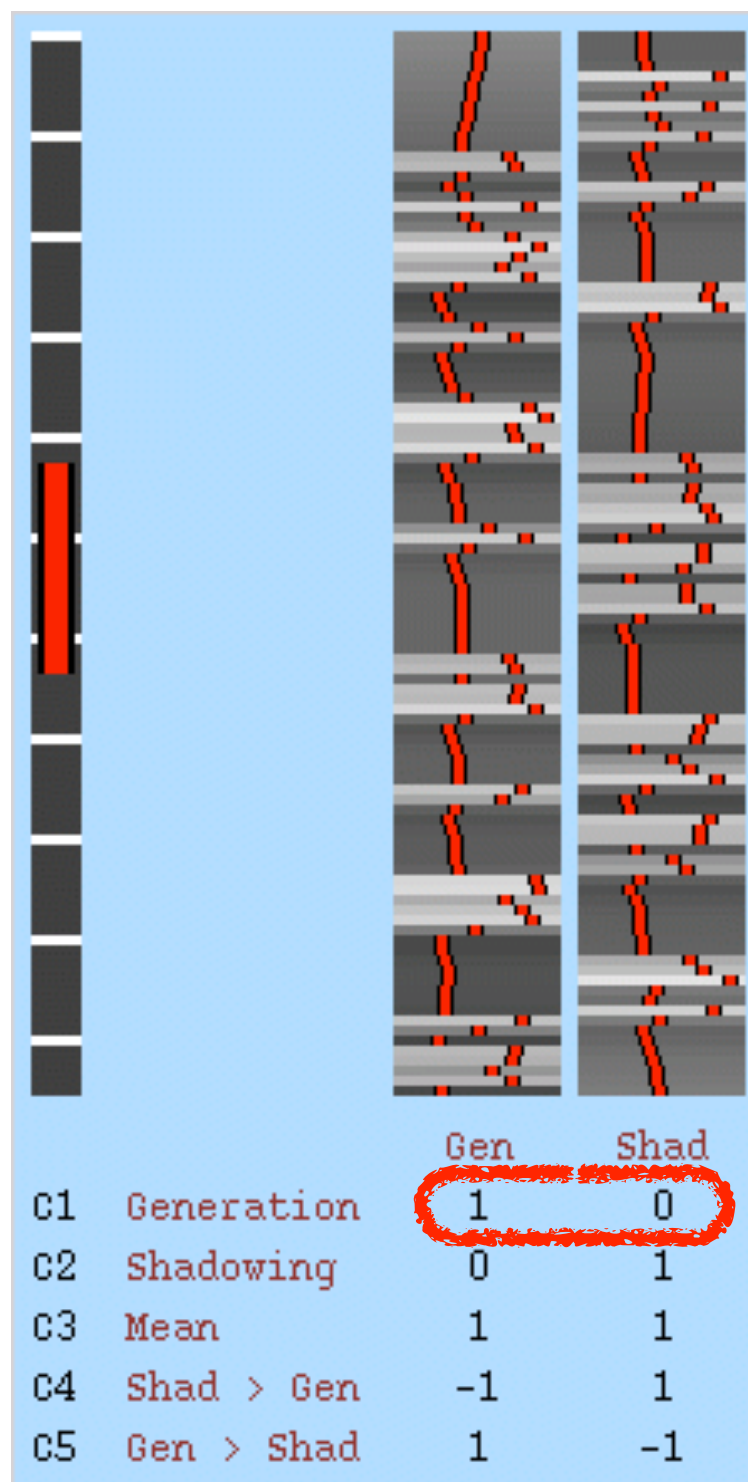
and the Residual Error





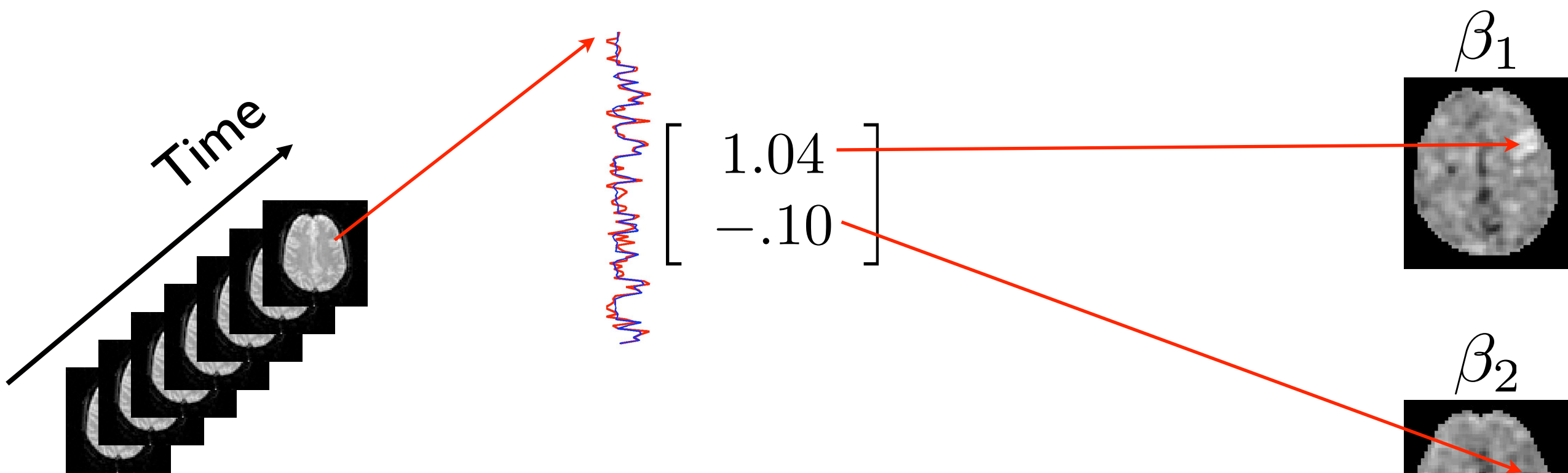
# *t*-contrasts

- $[1 \ 0]$  : EV1 only (i.e. Generation vs rest)
- $[0 \ 1]$  : EV2 only (i.e. Shadowing vs rest)



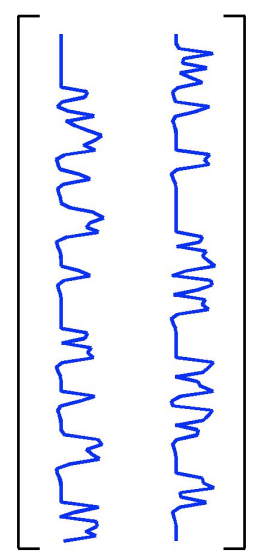


# t-contrasts



Contrast weight vector:  $\begin{bmatrix} 1 & 0 \end{bmatrix}$

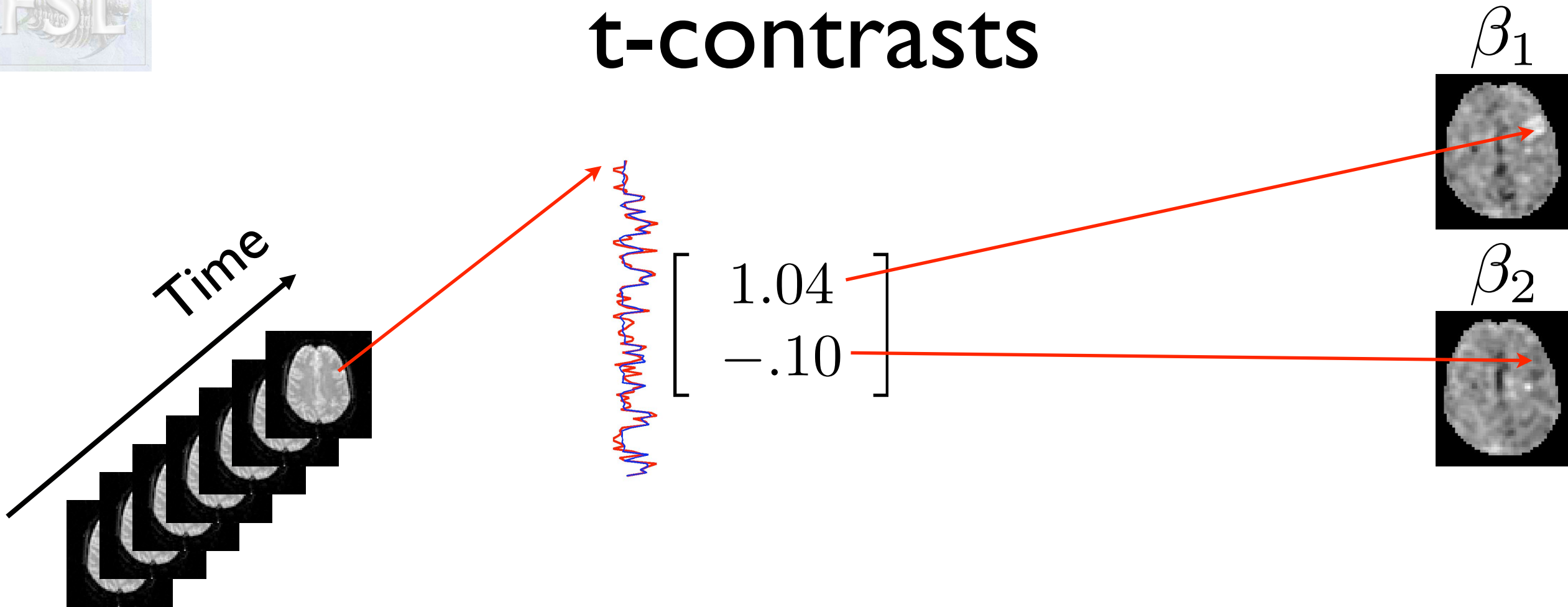
Asks the question: Where do we need this regressor to model the data, i.e. what parts of the brain are used when seeing nouns and generating related verbs?



Model

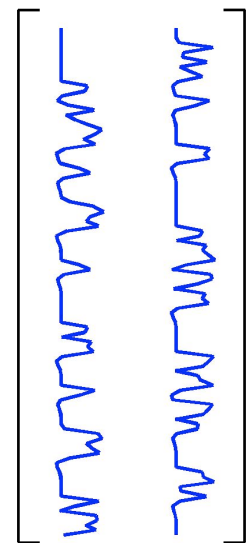


# t-contrasts



Contrast weight vector:  $\begin{bmatrix} 1 & 0 \end{bmatrix}$

$$\text{COPE} = 1 \times 1.04 + 0 \times -0.10 = 1.04$$

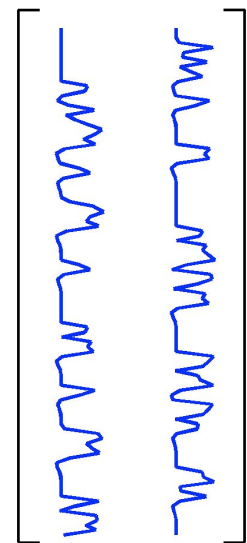
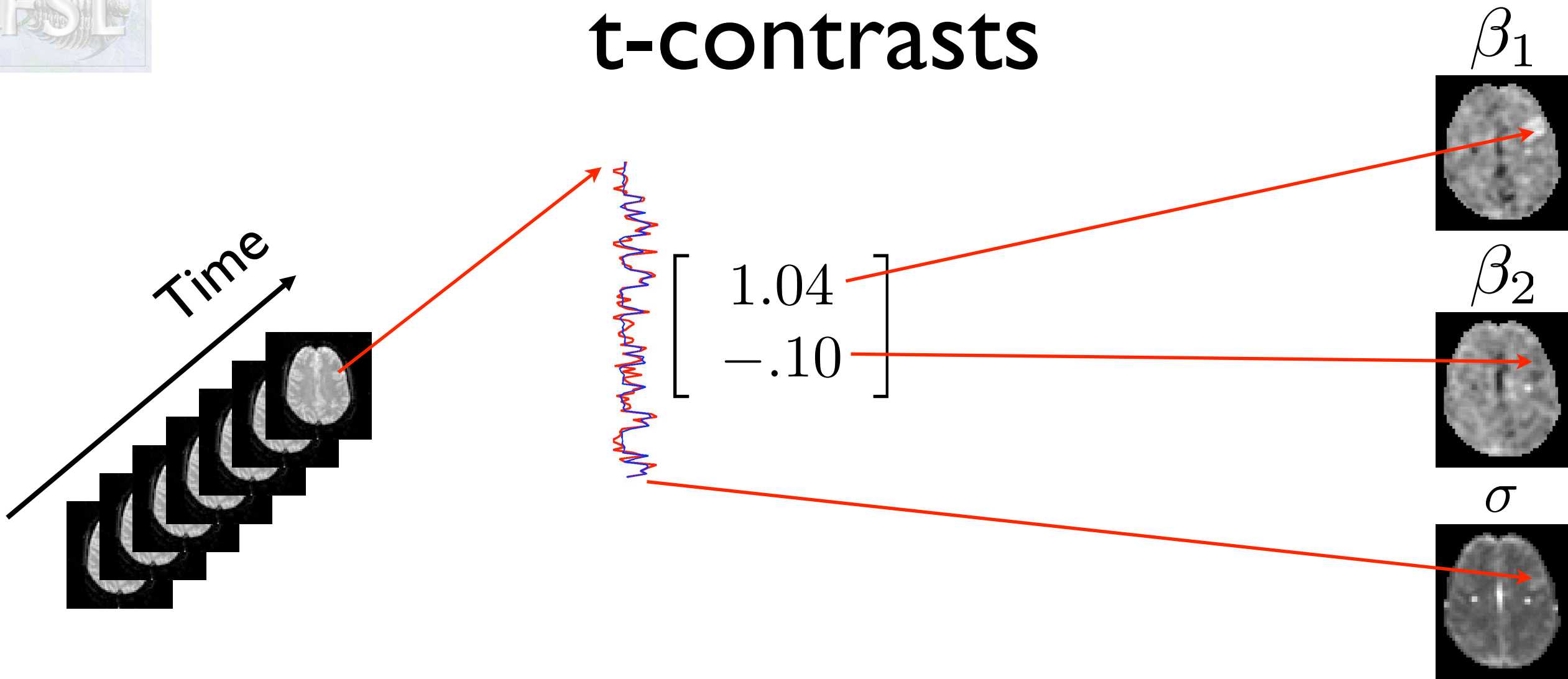


Model

$$\text{COPE} = \text{[Brain Slice]} = \beta_1$$



# t-contrasts



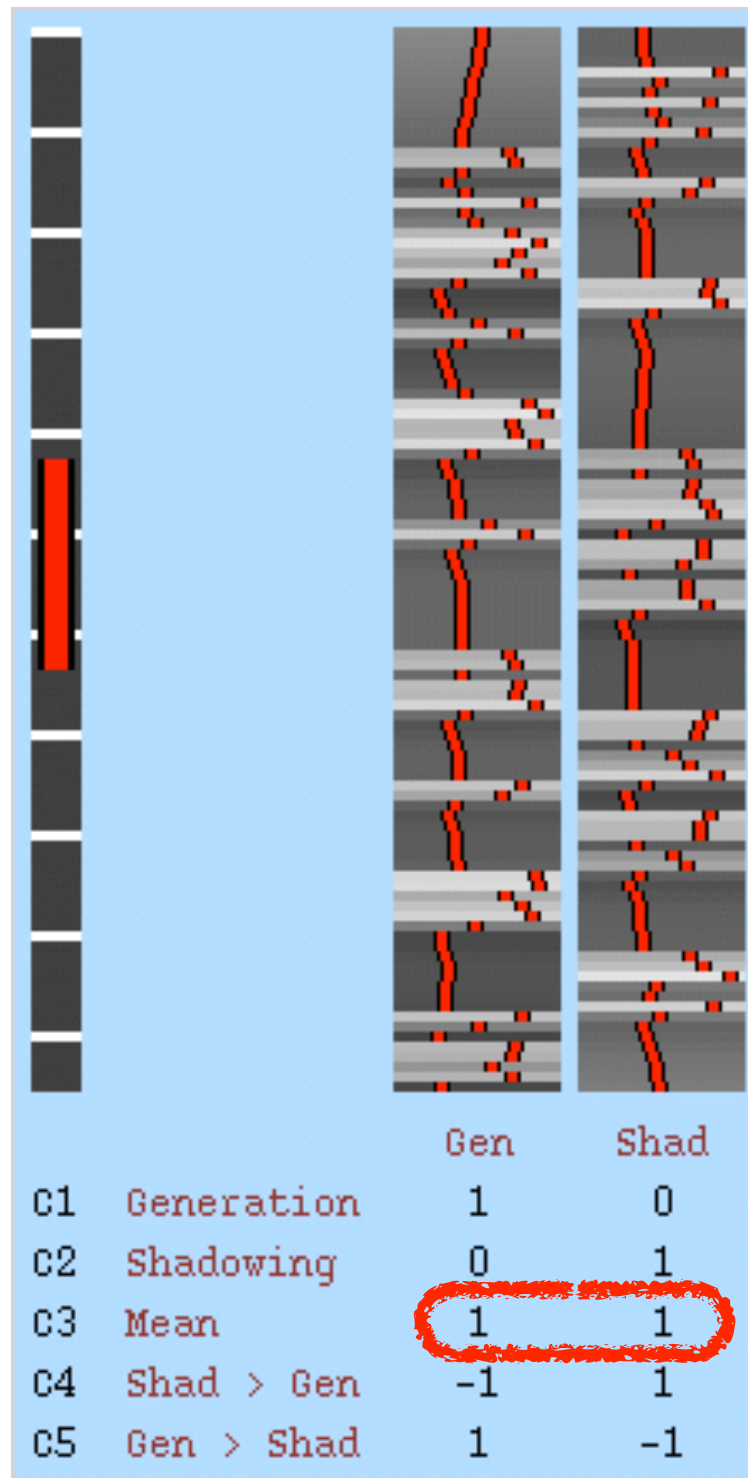
Model

$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{[Brain Slice } \beta_1 \text{]}}{\text{[Brain Slice } \sigma \text{]}} = \text{[Brain Slice } \beta_2 \text{]}$$



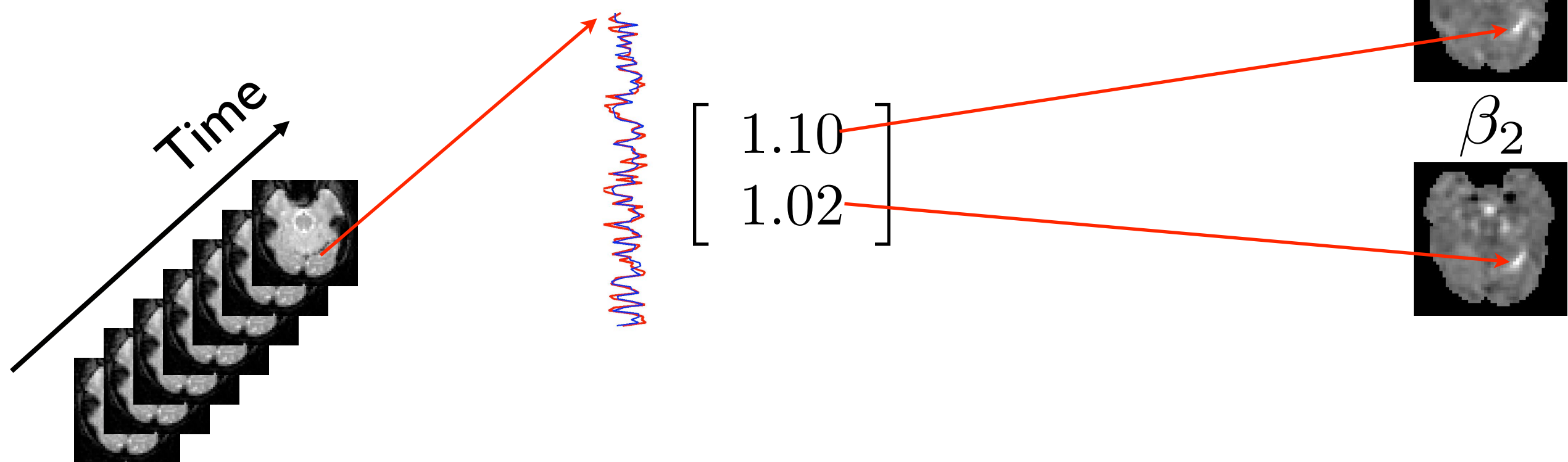
# *t*-contrasts

- $[1 \ 0]$  : EV1 only (i.e. Generation vs rest)
- $[0 \ 1]$  : EV2 only (i.e. Shadowing vs rest)
- $[1 \ 1]$  : EV1 + EV2 (Mean activation)



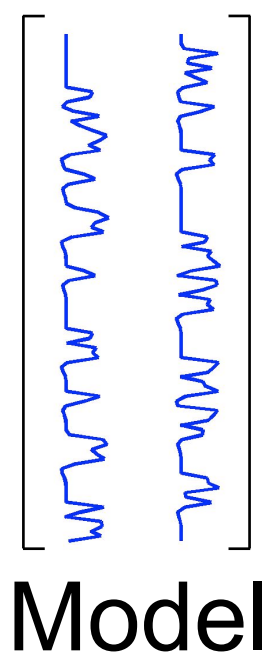


# t-contrasts



Contrast weight vector:  $[1 \ 1]$

$$\text{COPE} = 1 \times 1.10 + 1 \times 1.02 = 2.12$$

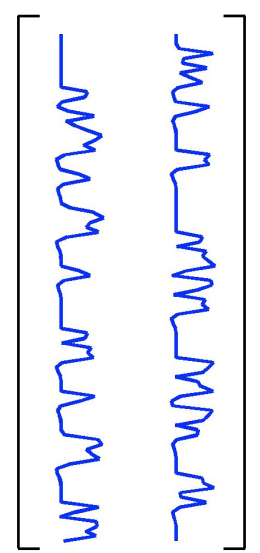
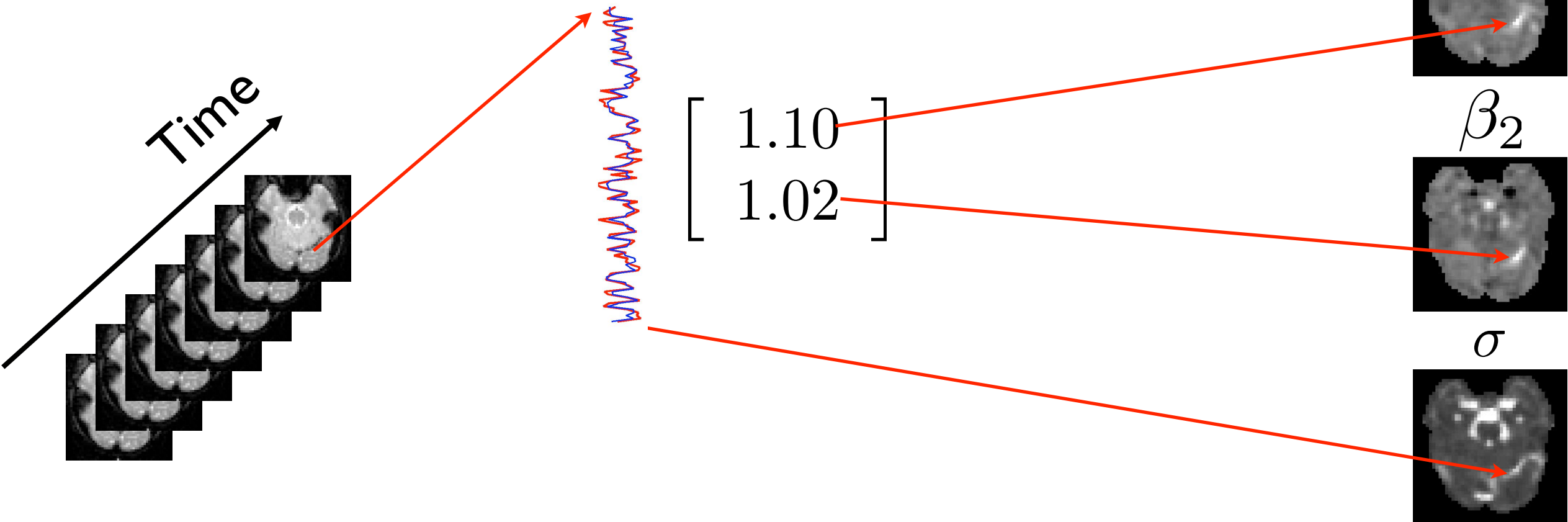


$$\text{COPE} = \text{[Brain Slice]} = \underset{\beta_1}{\text{[Brain Slice]}} + \underset{\beta_2}{\text{[Brain Slice]}}$$





# t-contrasts



Model

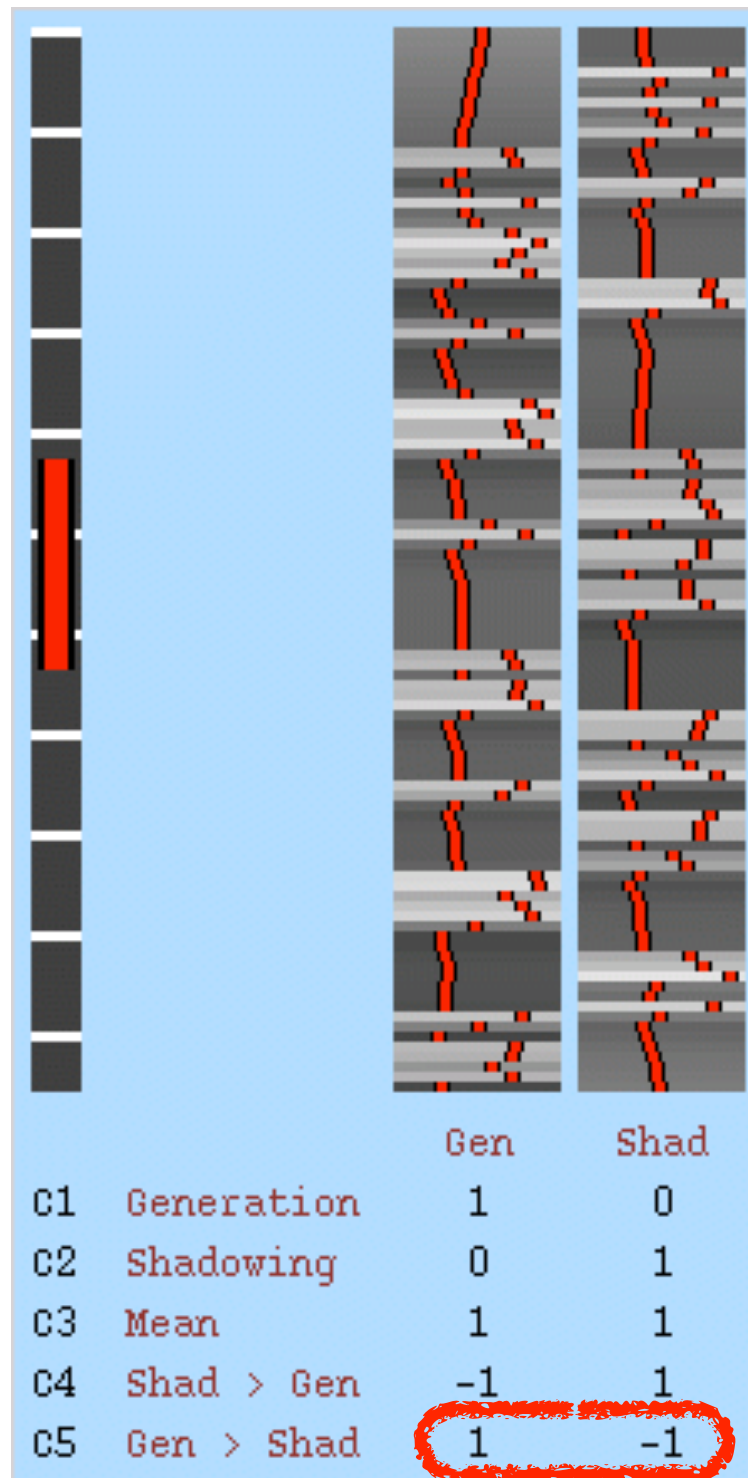
$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{Brain Slice } \beta_1}{\text{Brain Slice } \sigma} = \text{Brain Slice } t$$





# *t*-contrasts

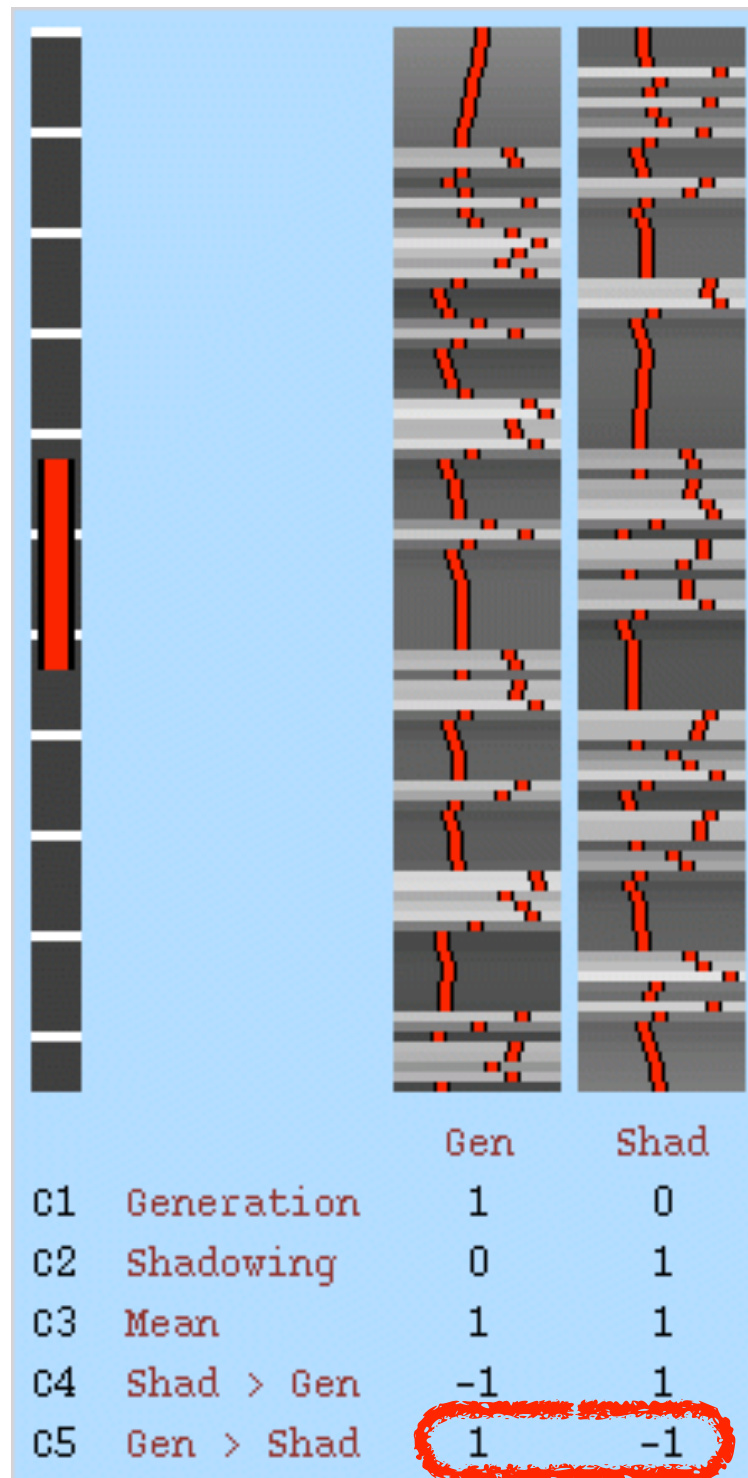
- $[1\ 0]$  : EV1 only (i.e. Generation vs rest)
- $[0\ 1]$  : EV2 only (i.e. Shadowing vs rest)
- $[1\ 1]$  : EV1 + EV2 (Mean activation)
- $[-1\ 1]$ : EV2 - EV1 (More activated by Shadowing than Generation)
- $[1\ -1]$ : EV1 - EV2 (More activated by Generation than Shadowing (*t*-tests are directional))





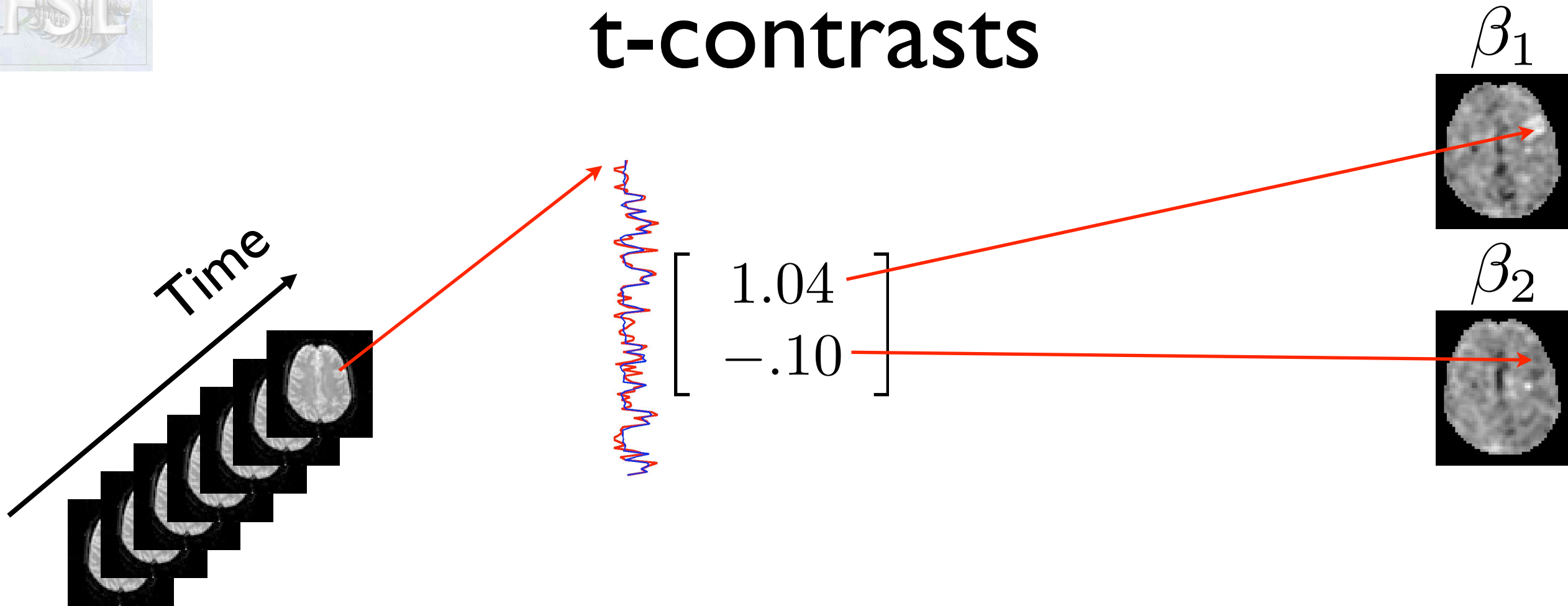
# *t*-contrasts

- $[1\ 0]$  : EV1 only (i.e. Generation vs rest)
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# t-contrasts



Contrast weight vector:  $\begin{bmatrix} 1 & -1 \end{bmatrix}$

$$\text{COPE} = 1 \times 1.04 - 1 \times -0.10 = 1.14$$

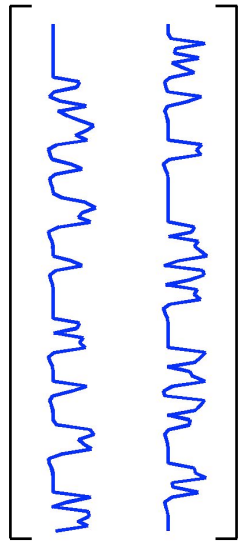
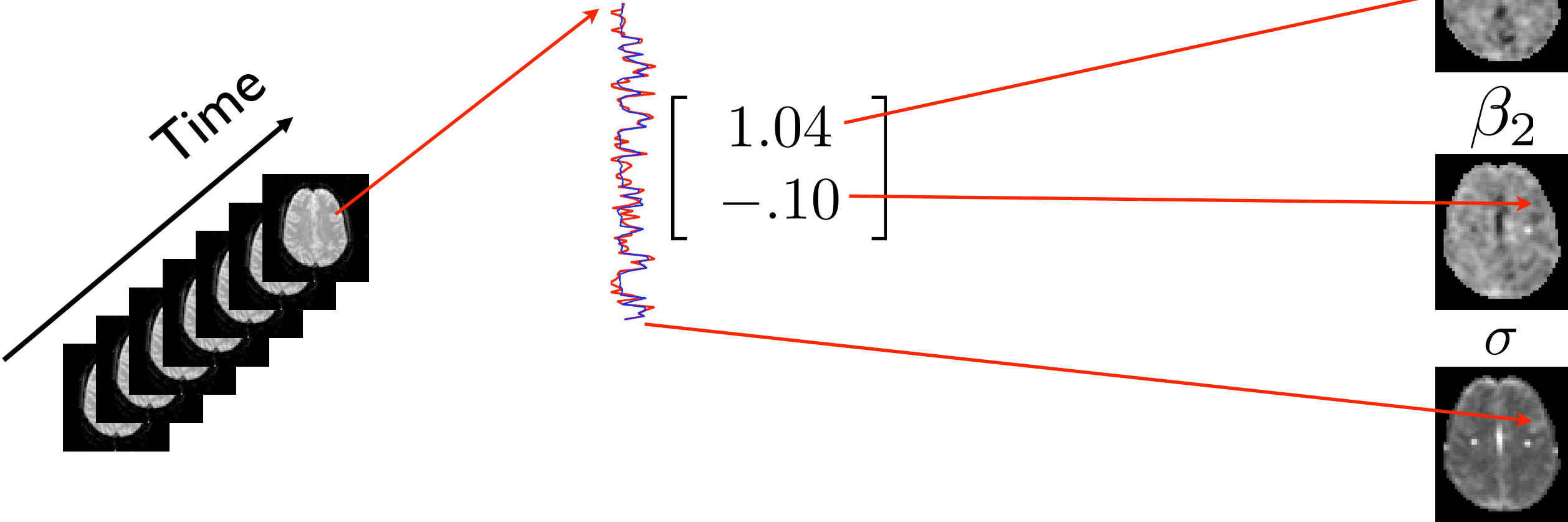
$$\begin{bmatrix} \text{Time Series 1} \\ \text{Time Series 2} \end{bmatrix}$$

Model

$$\text{COPE} = \text{Image} = \beta_1 - \beta_2$$



# t-contrasts

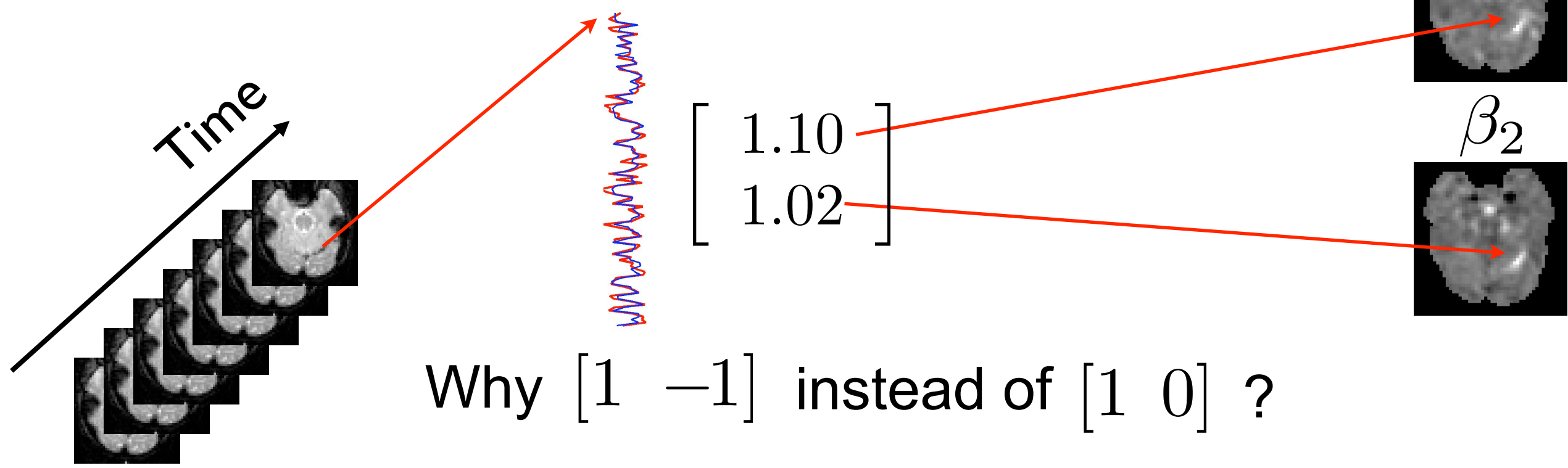


Model

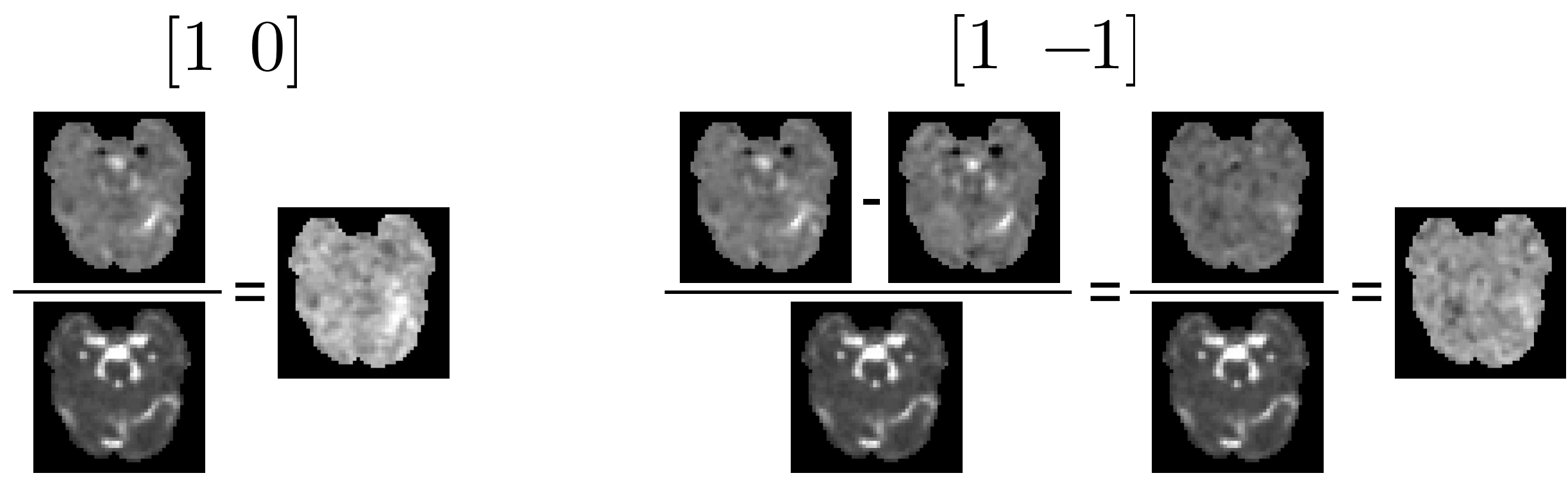
$$t = \frac{\text{COPE}}{\text{std}(\text{COPE})} = \frac{\text{Brain Slice } \beta_1}{\text{Brain Slice } \sigma} = \text{Brain Slice } \beta_2$$



# t-contrasts



Why  $\begin{bmatrix} 1 & -1 \end{bmatrix}$  instead of  $\begin{bmatrix} 1 & 0 \end{bmatrix}$  ?



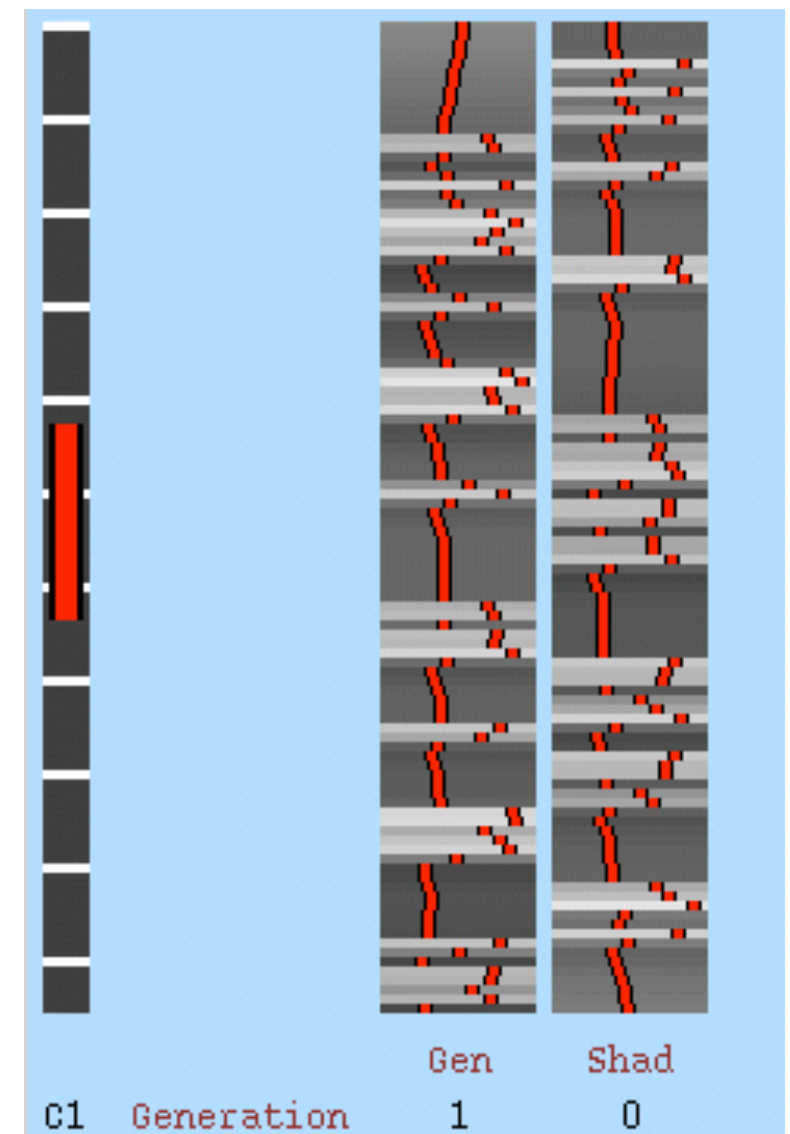




# F-contrasts

We have two conditions:  
Word Generation and Shadowing

We want to know:  
Is there an activation to any condition?



First we ask: Is there activation to Generation?

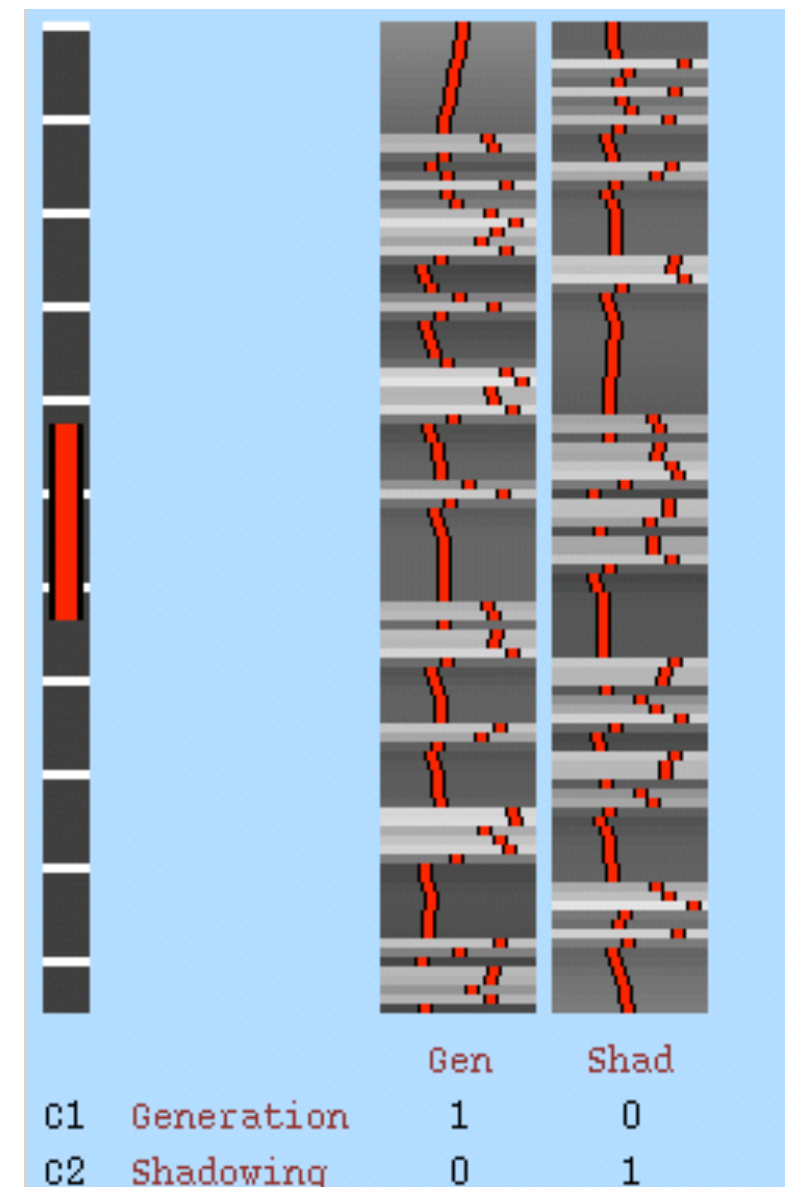
$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$



# F-contrasts

We have two conditions:  
Word Generation and Shadowing

We want to know:  
Is there an activation to any condition?



Then we ask: Is there activation to Shadowing?

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$





# F-contrasts

We have two conditions:  
Word Generation and Shadowing

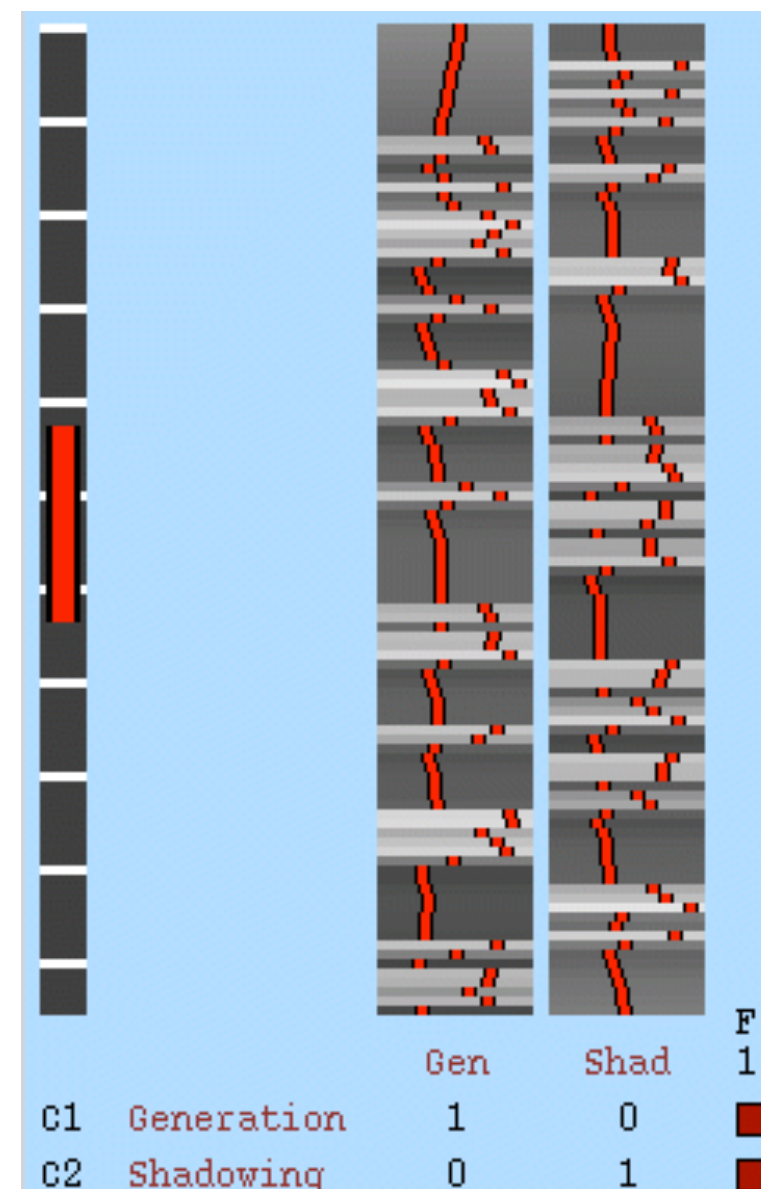
We want to know:  
Is there an activation to any condition?

EVs Contrasts & F-tests

Setup contrasts & F-tests for Original EVs

Contrasts  F-tests

		Title	EV1	EV2	F1
OC1	<input checked="" type="checkbox"/>	Generation	<input type="text" value="1"/>	<input type="text" value="0"/>	<input checked="" type="checkbox"/>
OC2	<input checked="" type="checkbox"/>	Shadowing	<input type="text" value="0"/>	<input type="text" value="2"/>	<input checked="" type="checkbox"/>



Then we add the OR

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



# F-contrasts

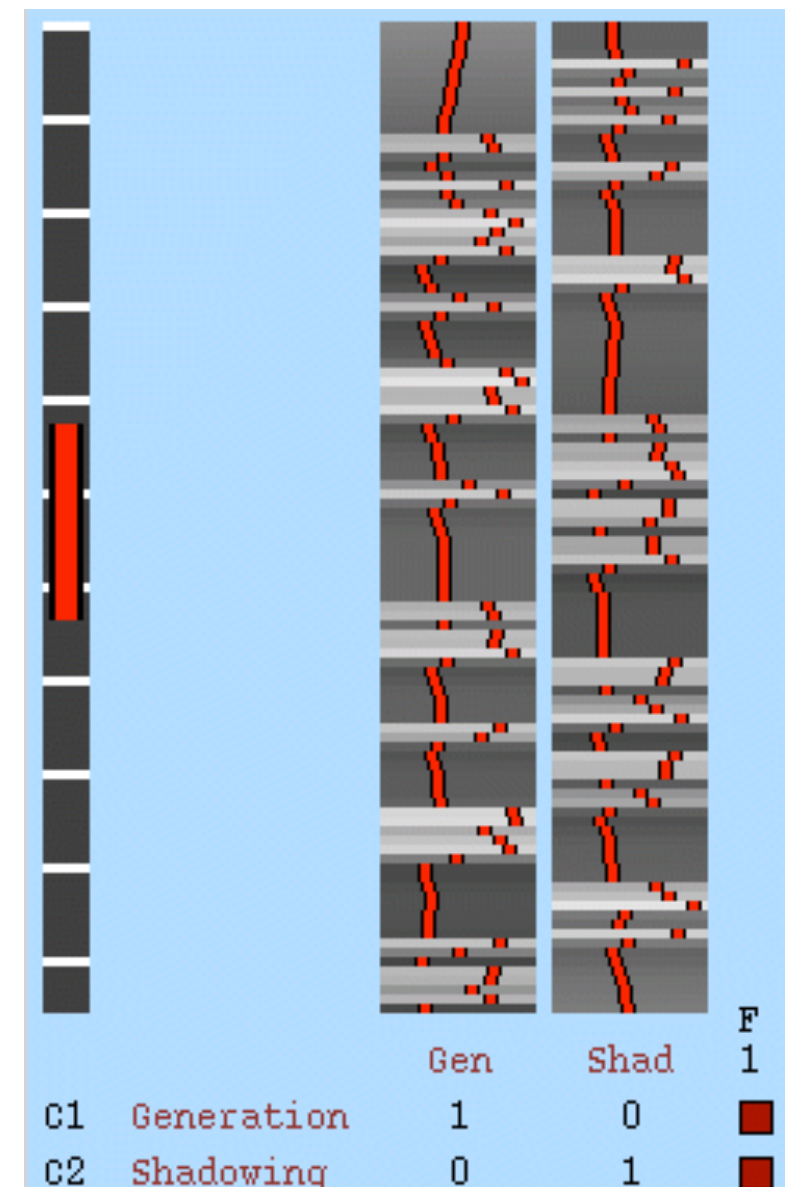
We have two conditions:  
Word Generation and Shadowing

We want to know:  
Is there an activation to any condition?

Is there an activation to any condition?

Is equivalent to:

Does any regressor explain the  
variance in the data?

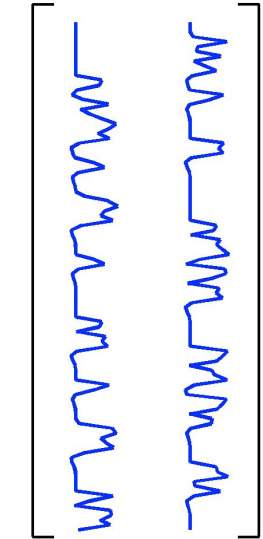


Then we add the OR

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



# F-contrasts



Full Model

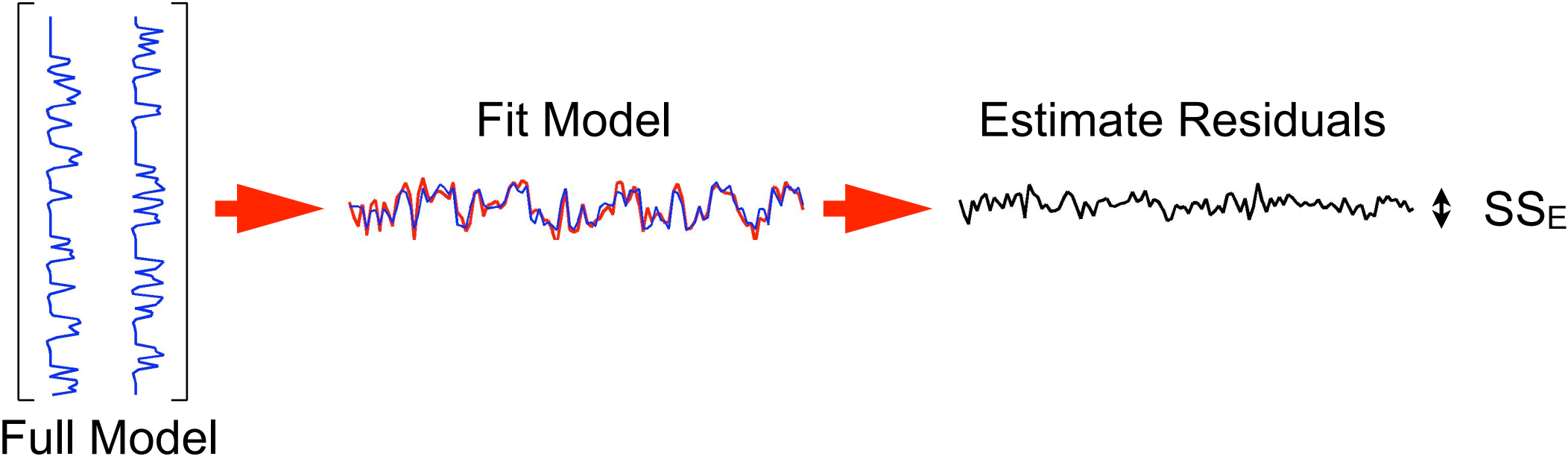
&

Data





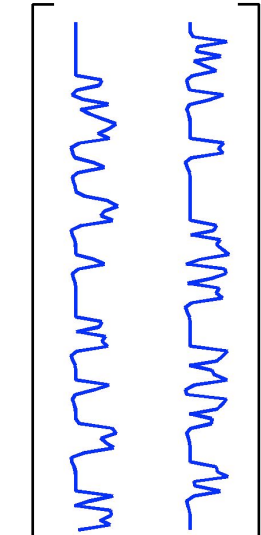
# F-contrasts



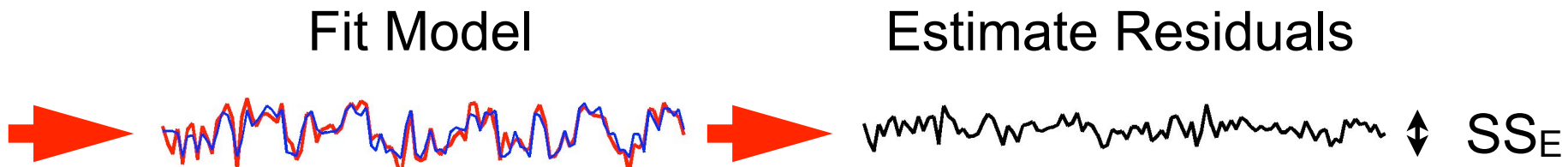


# F-contrasts

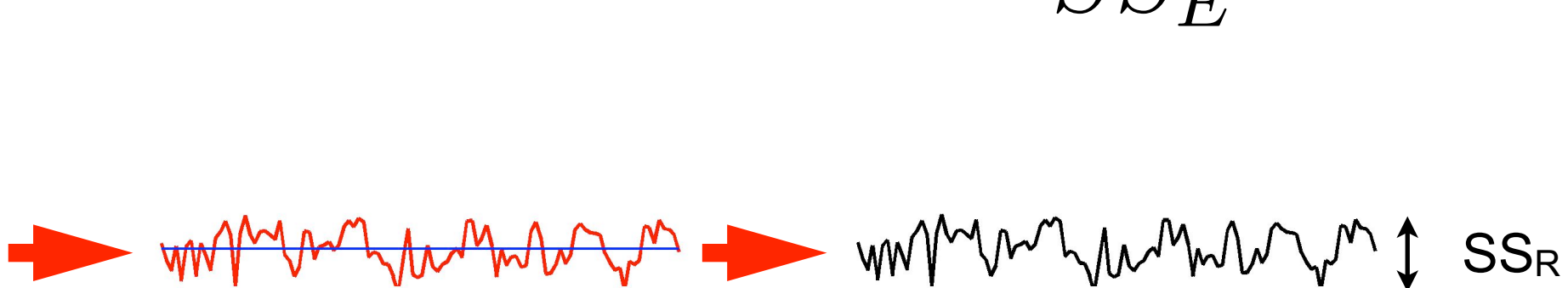
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Full Model



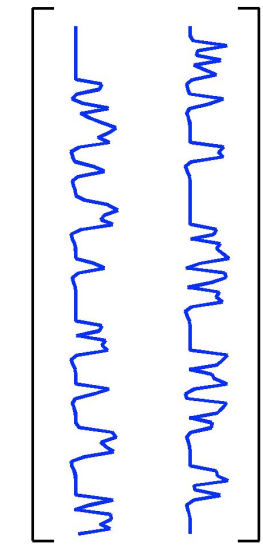
$$F = \frac{SS_R - SS_E}{SS_E} = \frac{\updownarrow - \updownarrow}{\updownarrow}$$



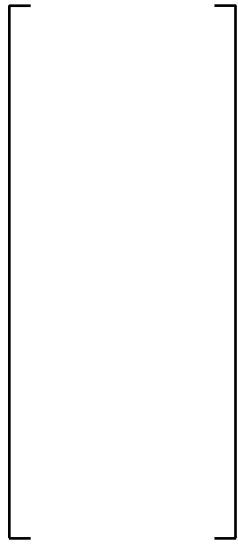
Reduced Model



# F-contrasts



Full Model



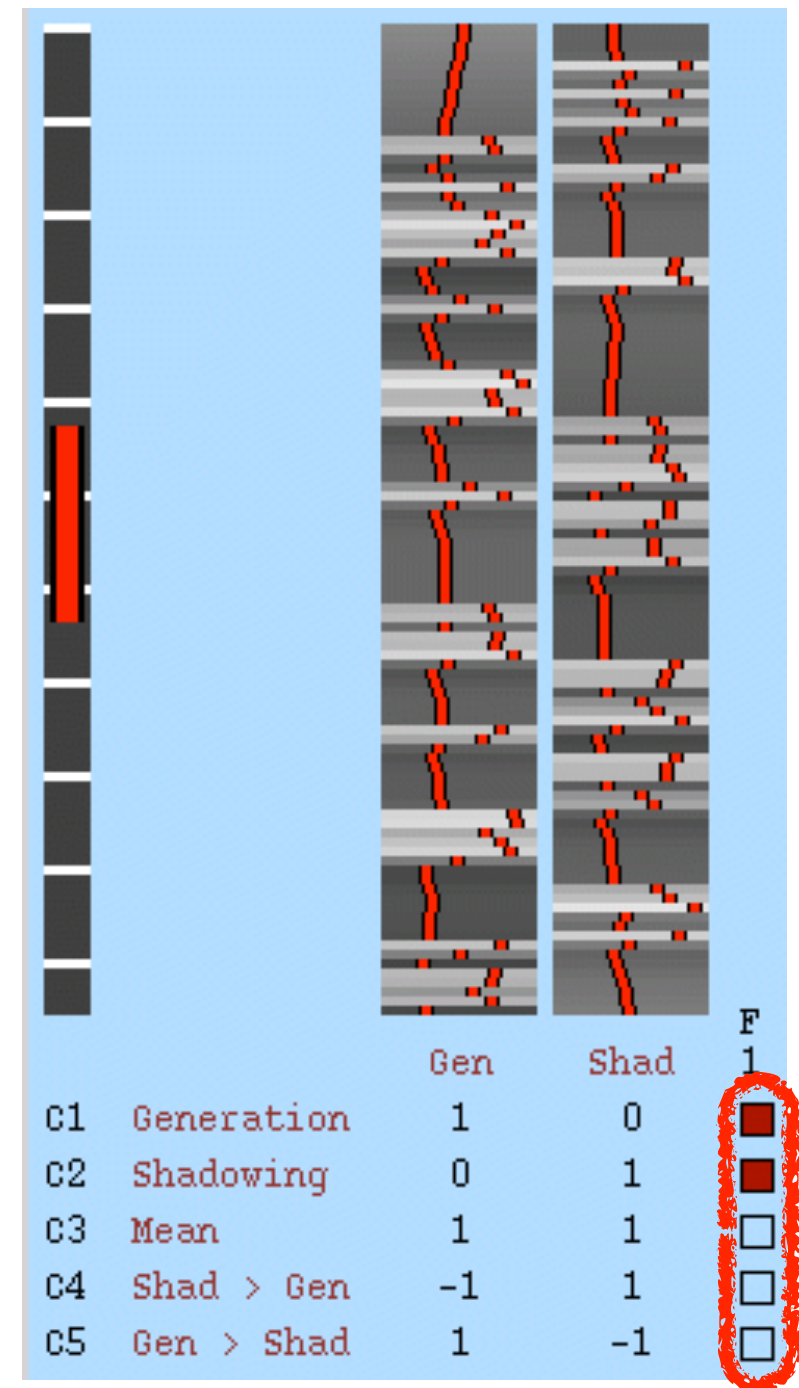
Reduced Model

$$F = \frac{SS_R - SS_E}{SS_E} = \frac{\text{Brain A} - \text{Brain B}}{\text{Brain C}} = \frac{\text{Brain D}}{\text{Brain E}} = \text{Brain F}$$



# F-contrasts

- Two conditions: A, B
- Is any condition significant?
- Set of COPEs form an F-contrast
- Or: “Is there a significant amount of power in the data explained by the combination of the COPEs in the F-contrast?”
- F-contrast is F-distributed



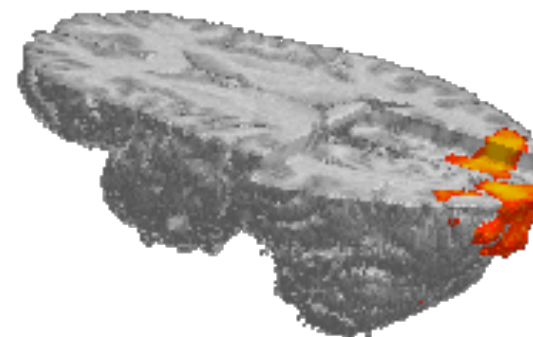




# Summary

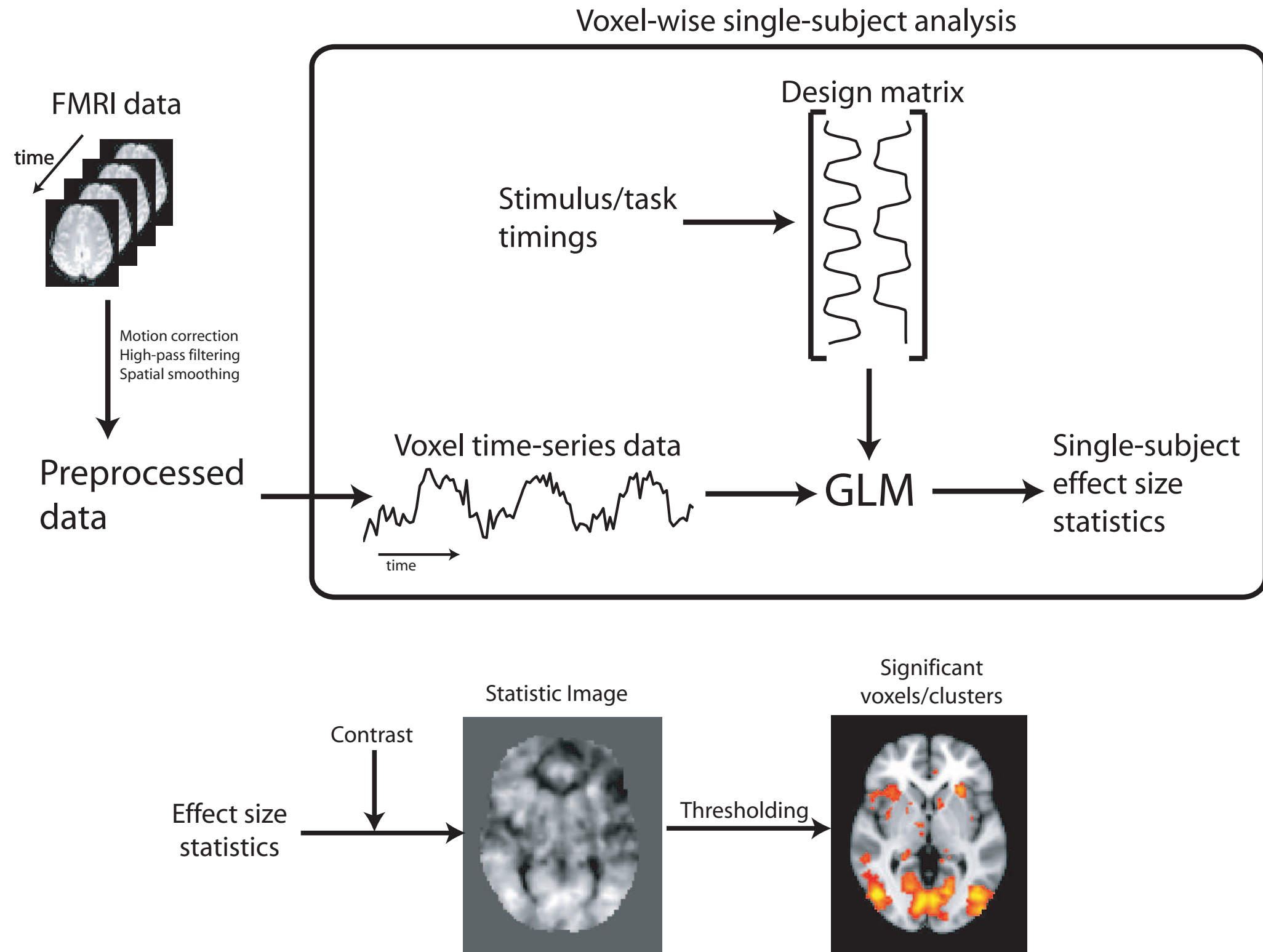
- The GLM is used to summarise data in a few parameters that are pertinent to the experiment.
- GLM predicts how BOLD activity might change as a result of the experiment.
- We can test for significant effects by using t or f contrasts on the GLM parameters

That's all folks





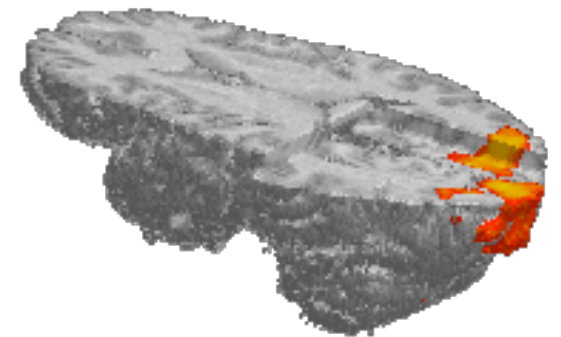
# Single-Session Analysis





# FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- T and F Contrasts
- **Null hypothesis testing**
- The residuals
- Thresholding: multiple comparison correction



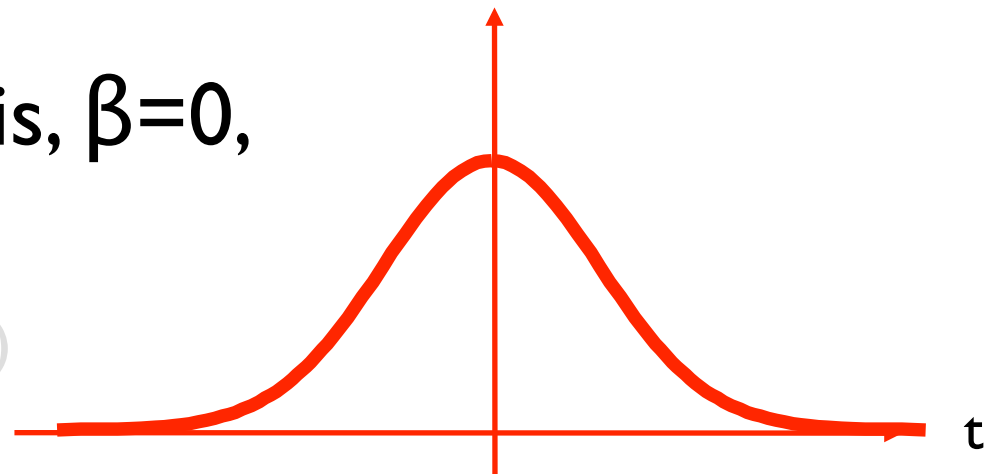


# Null Hypothesis Testing

t-statistic:  $t = \frac{\hat{\beta}}{\widehat{std(\beta)}}$

Under null hypothesis,  $\beta=0$ ,  
t is t-distributed

(what are the chances of that?)

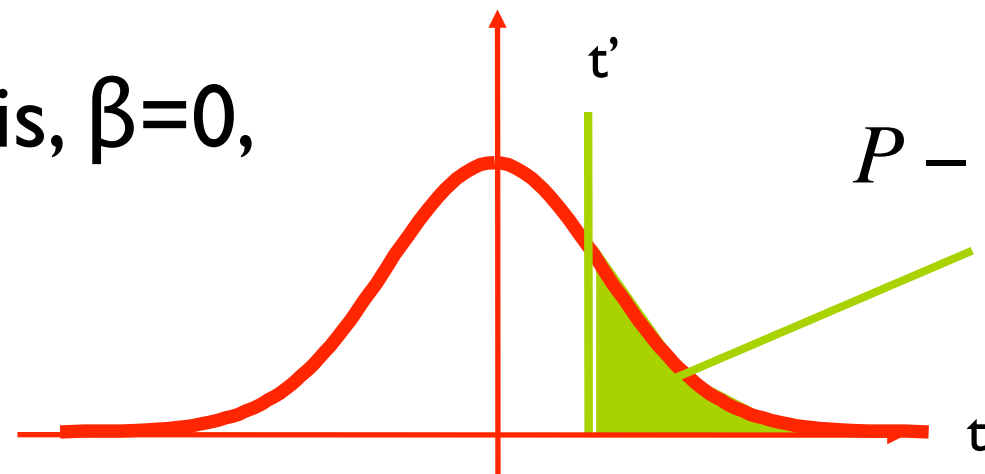




# Null Hypothesis Testing

t-statistic:  $t = \frac{\hat{\beta}}{\widehat{std}(\beta)}$

Under null hypothesis,  $\beta=0$ ,  
t is t-distributed



$$P\text{-Value} = p(t > t' | \beta = 0)$$

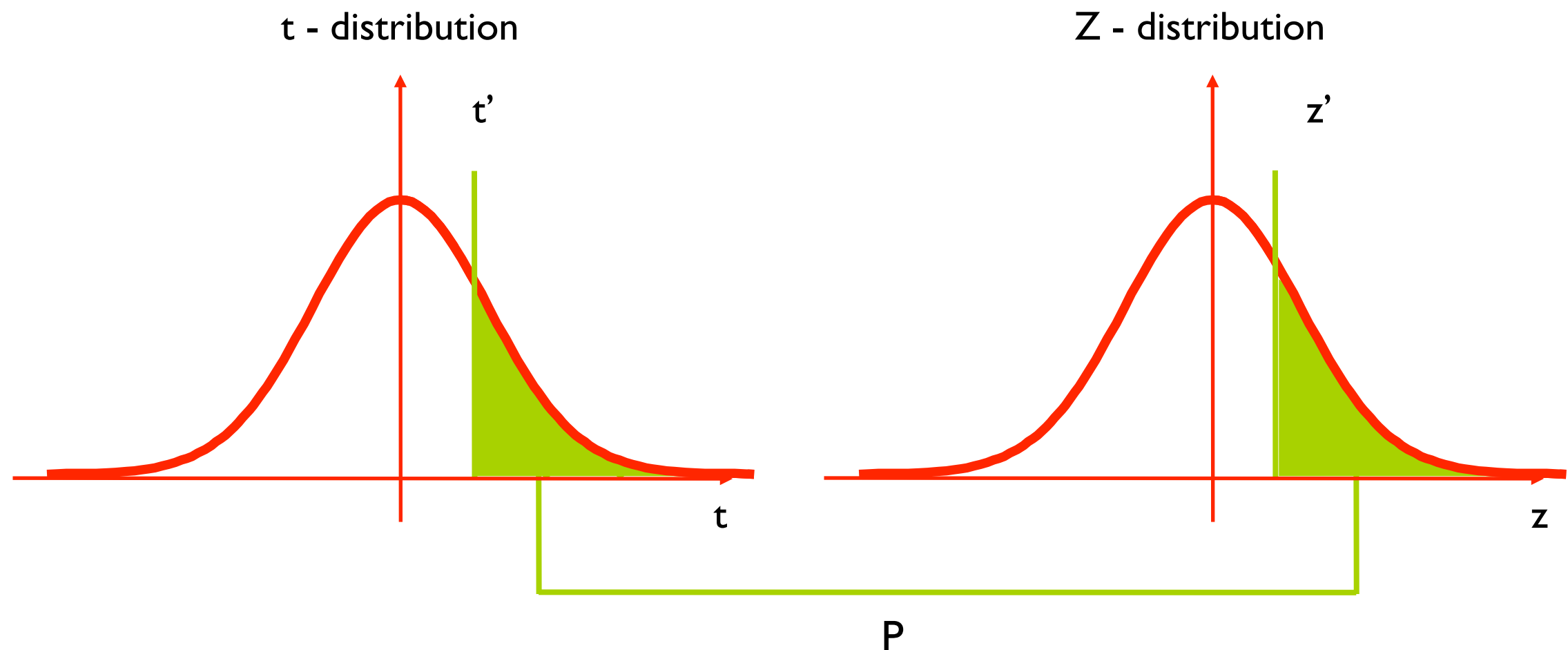
Small P-Value = null hypothesis unlikely

If P-Value < P-threshold then voxel is “active”

P-threshold corresponds to False Positive Rate (FPR)



# T to P to Z

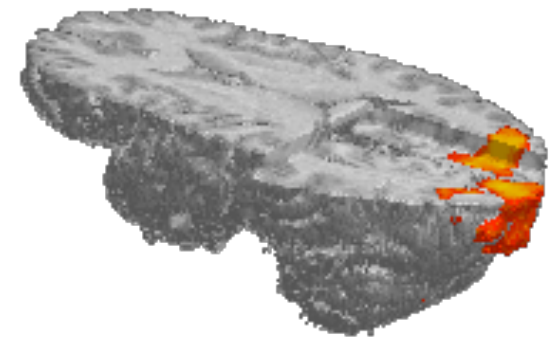


- FEAT performs spatial inference on z statistic maps
- Therefore, we convert t statistics to z statistics by equating probabilities under the tails of the distributions ( $t' \rightarrow p \rightarrow z'$ )



# FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- **The residuals**
- Thresholding: multiple comparison correction







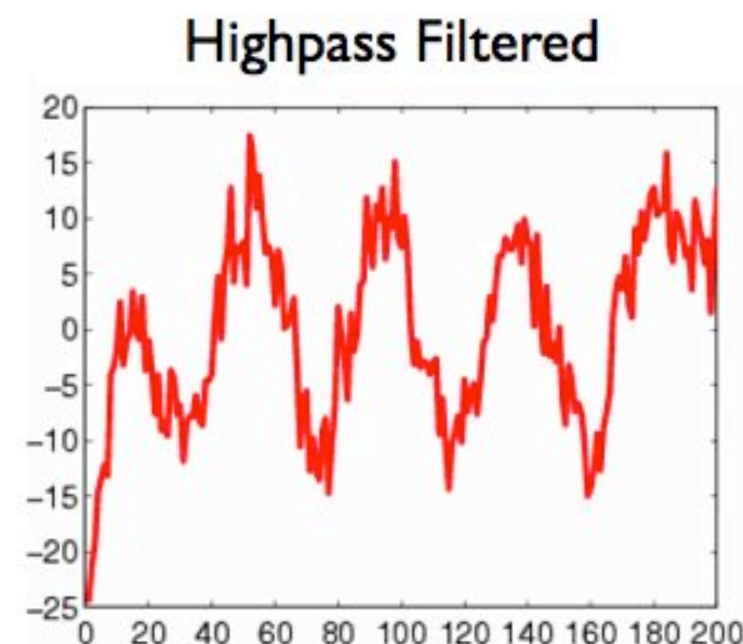
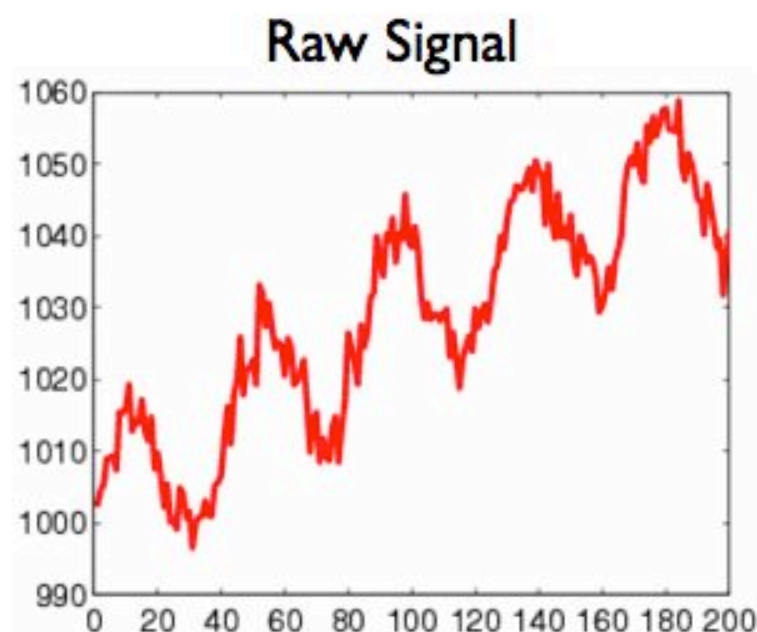
# Choosing High-Pass Filter Cut-off

- Can use the tool *cutoffcalc* to determine a good cut-off value

Remember that  
MJ mentioned  
highpass filtering?



## Temporal Filtering: Highpass



- Removes low frequency signals, including linear trend
- Must choose cutoff frequency carefully (lower than frequencies of interest = longer period)

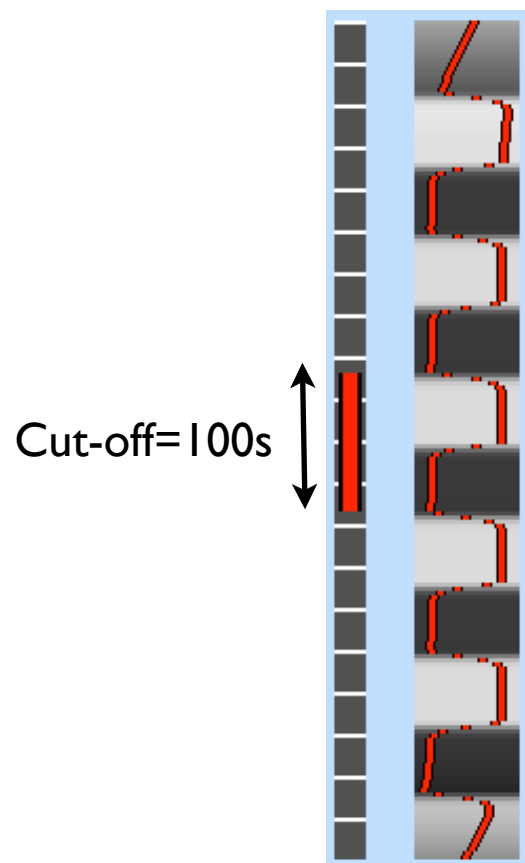


# Choosing High-Pass Filter Cut-off

- Can use the tool *cutoffcalc* to determine a good cut-off value  
**OR**
- Set by hand, but make sure model is not badly affected

Example: Boxcar EV with period

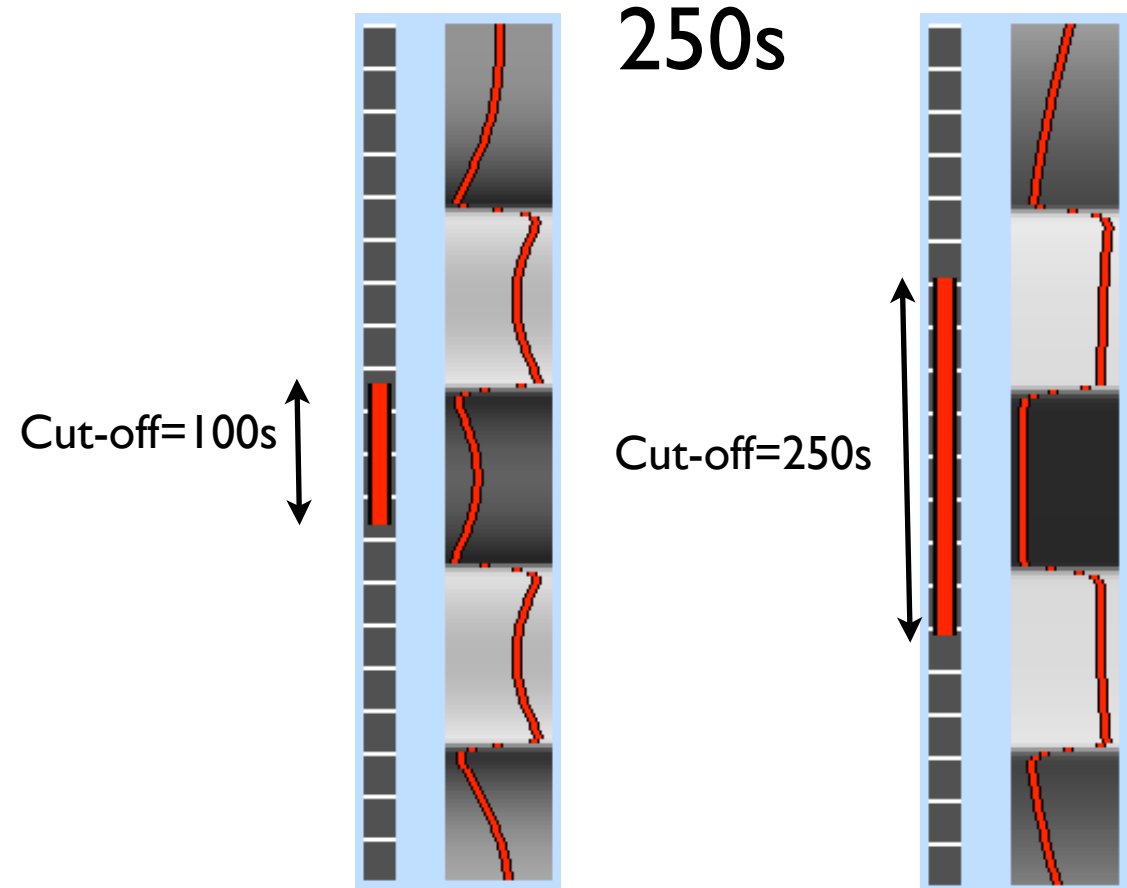
100s



Negligible effect on  
EV, so use cut-off of  
100s

Example: Boxcar with period

250s



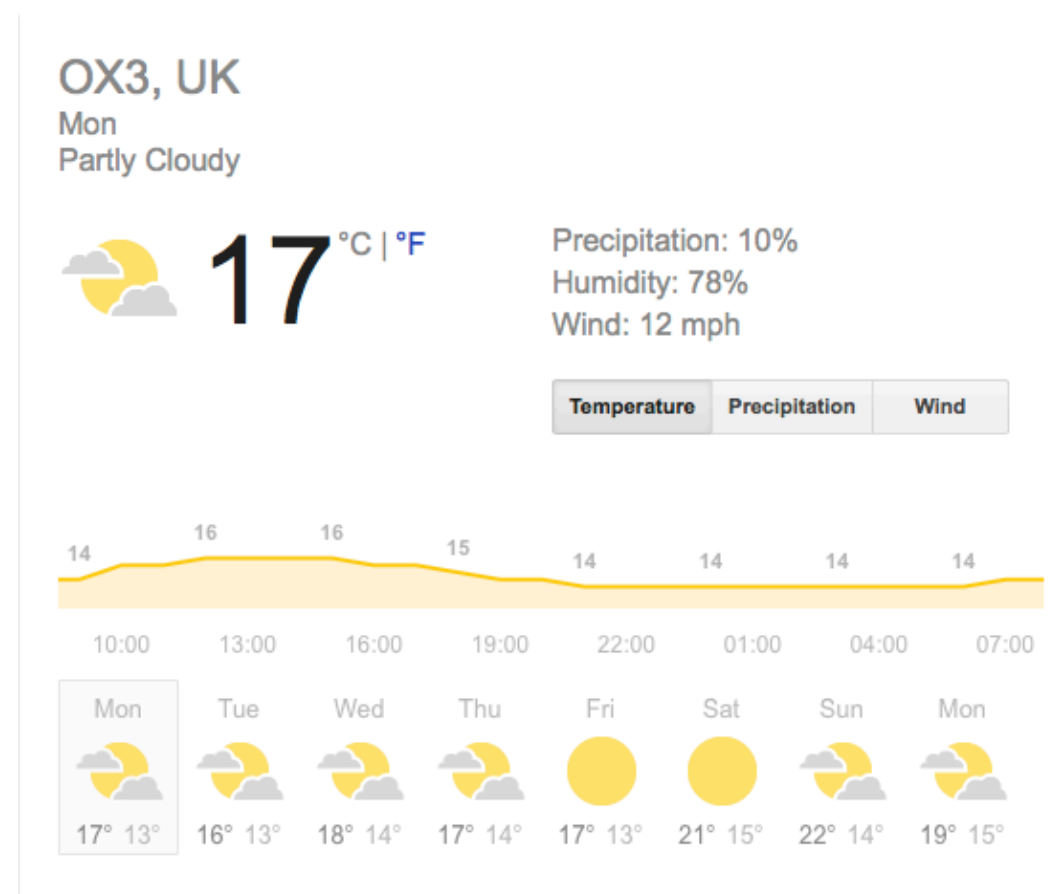
Substantial effect  
on EV, so need  
longer cut-off

Cut-off=250s

Negligible effect on  
EV, so use cut-off of  
250s



# Non-independent/Autocorrelation/ Coloured FMRI noise



Uncorrected, this causes:

- biased stats (increased false positives)
- decreased sensitivity

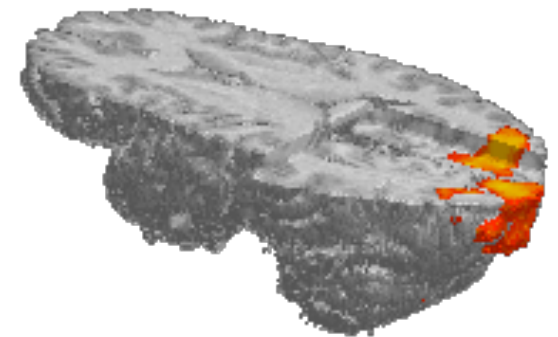
FSL fixes it for you in FEAT!

Cannot use randomise (see later) because of autocorrelation



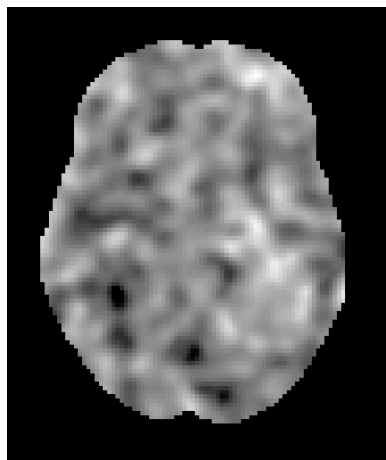
# FMRI Modelling and Statistics

- An example experiment
- Multiple regression (GLM)
- T and F Contrasts
- Null hypothesis testing
- The residuals
- **Thresholding: multiple comparison correction**

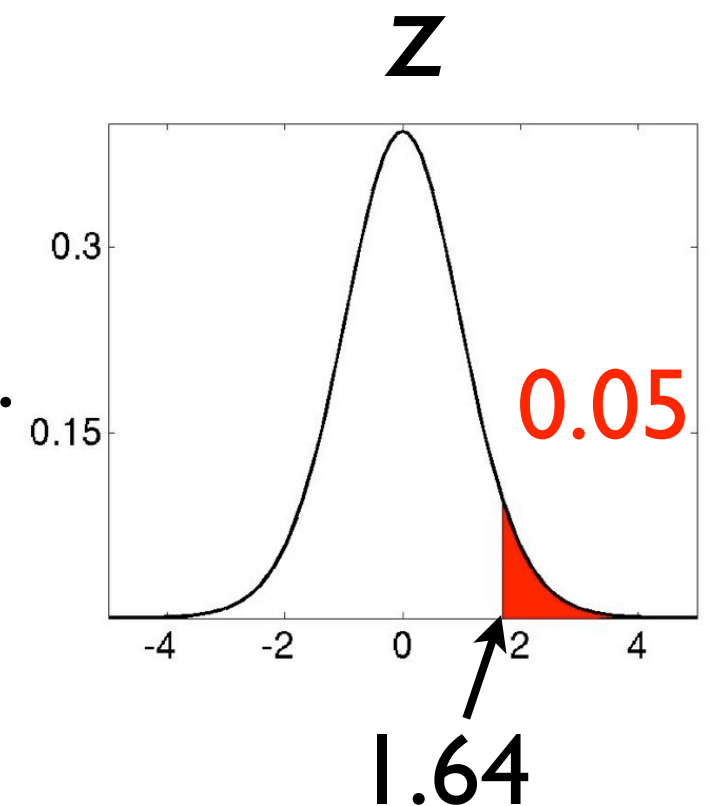




# What happens when we apply “standard” statistical testing to imaging data?



z-map where each voxel  $\sim N$ .  
Null-hypothesis true everywhere, i.e.  
NO ACTIVATIONS



z-map  
thresholded at  
1.64



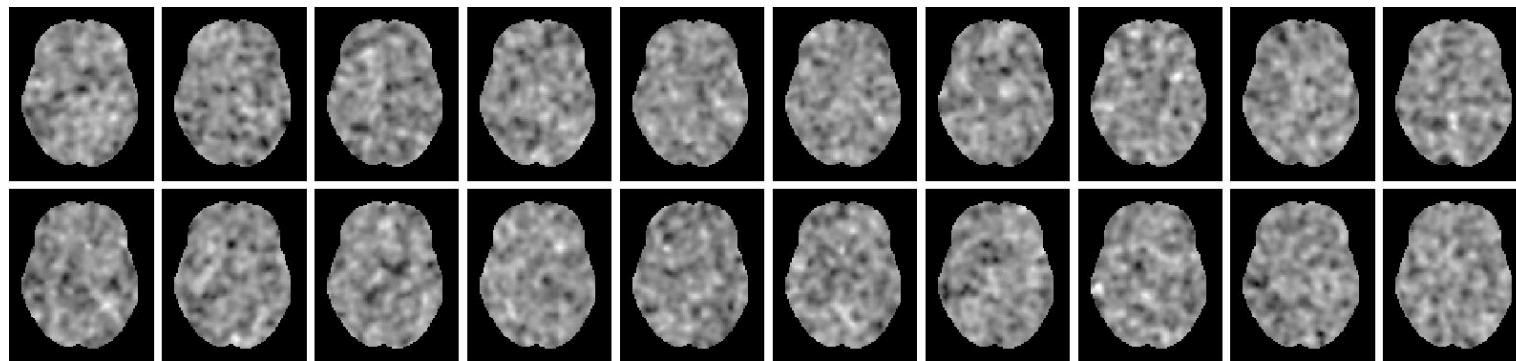
16 clusters  
288 voxels  
~5.5% of the voxels

That's a LOT of false positives



# What we really want

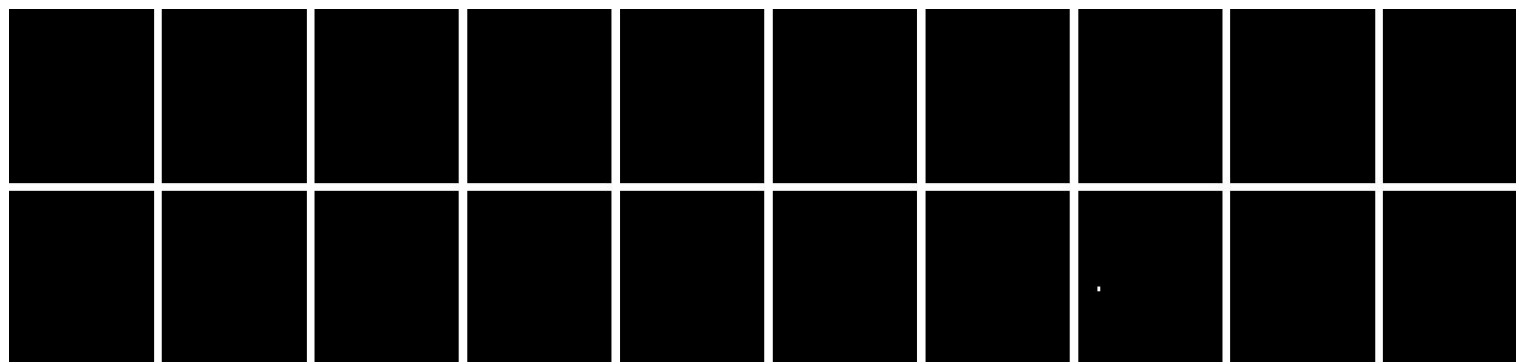
Let's say we perform a series of identical studies



Each z-map is the end result of a study

Let us further say that the null-hypothesis is true

We want to threshold the data so that only once in 20 studies do we find a voxel above this threshold



There will be a whole talk on how to find such a threshold





# Summary

- We test for an effect through a null-hypothesis, that we might reject.
- The null-hypothesis is rejected if the observed statistic is “too unlikely”.
- When thresholding the number of false positives needs to be controlled across the entire brain

That's all folks

