



# Advanced GLM designs



# Advanced Analysis: Parametric Designs

## Scenario:

Interested in specific responses to multiple levels of a painful stimulus

## Specific questions:

Are there regions showing significant responses to painful stimuli?

Are there regions where higher intensity stimuli produce larger responses?

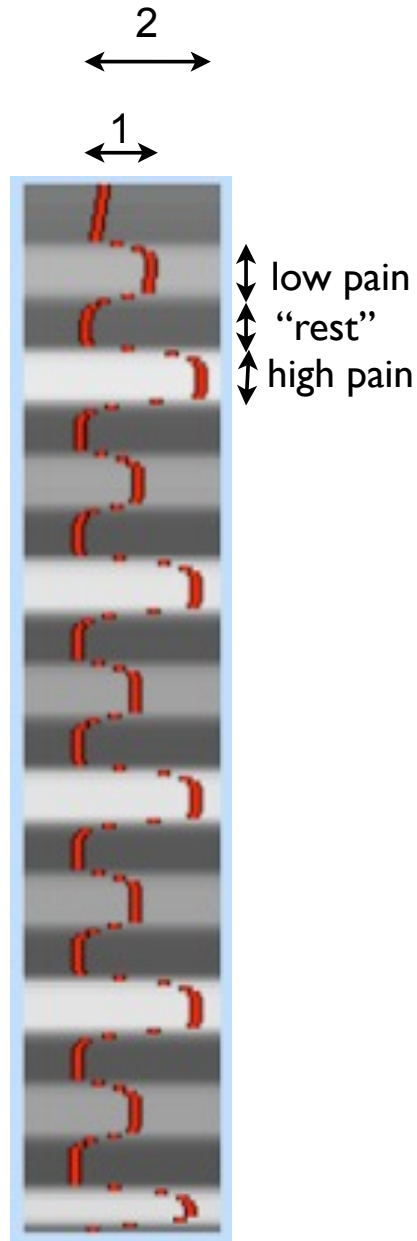
Are there regions with a linear response across multiple levels of stimuli?

## Solution:

Multiple regressors

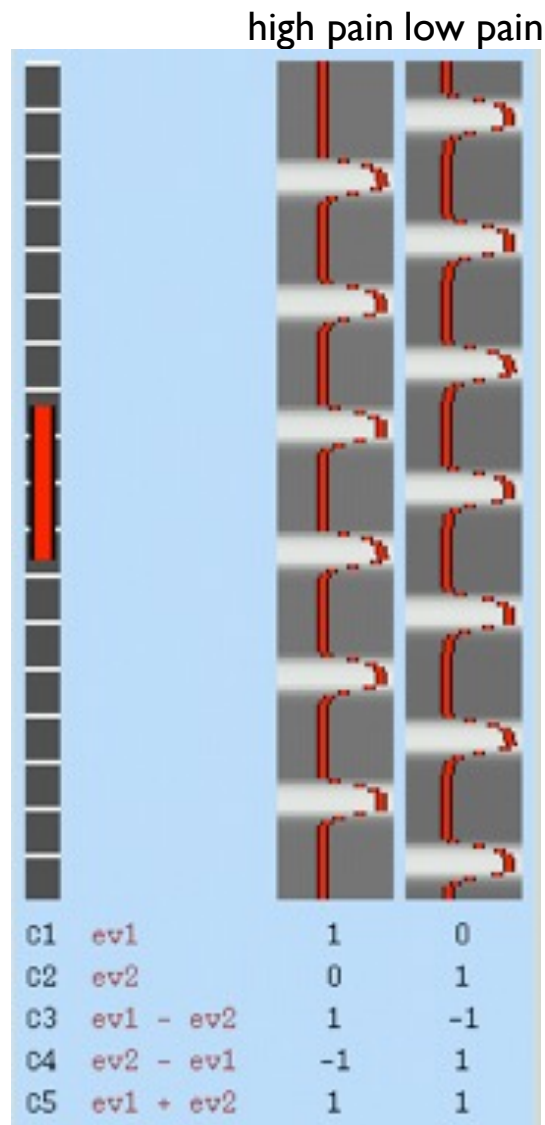
Contrasts and F-tests

# Analysis of responses to multiple levels of painful stimuli: modelling



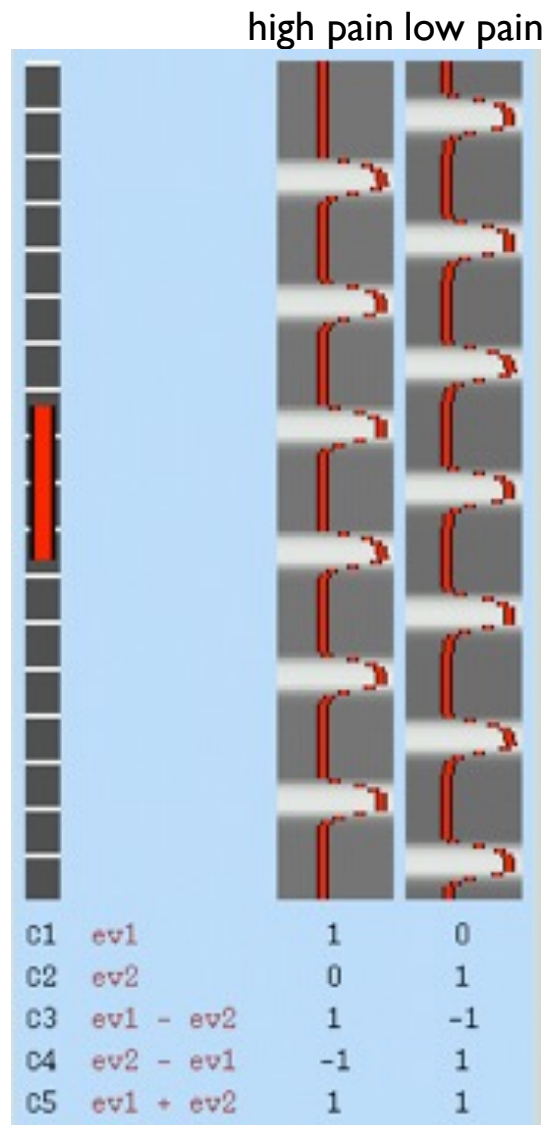
- Possible approach: model a specific hypothesis - high produces twice the response as low
- Pre-supposes relationship between stimulation strength and response
- Can only ask the question about the pre-supposed relationship

# Analysis of responses to multiple levels of painful stimuli: modelling



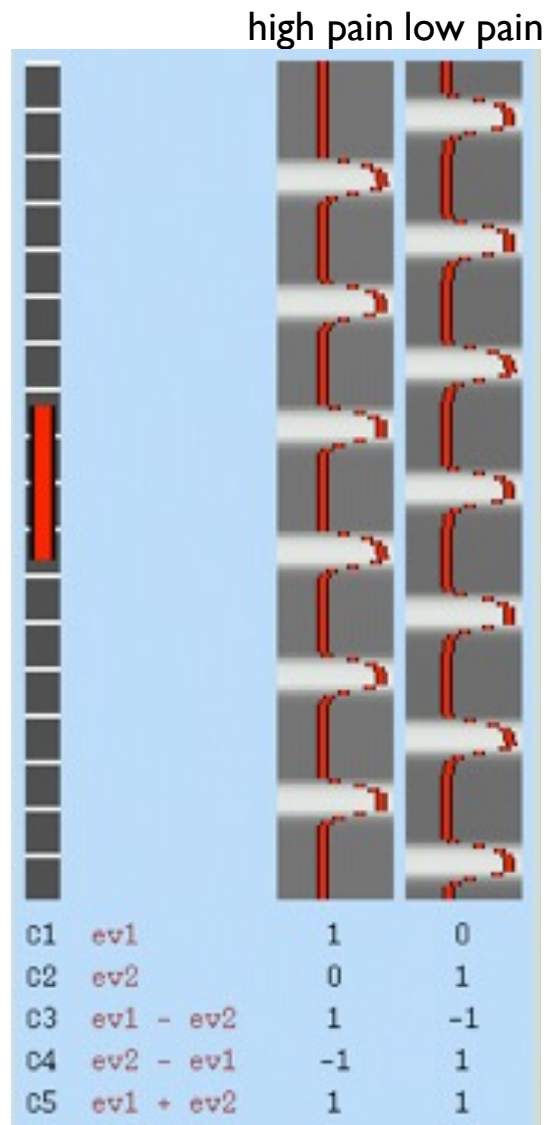
- Better approach: model as if two completely different stimuli
- Now, no pre-supposition about relationship between stimulation strength and response
- Can assess responses to individual stimuli
  - t-contrast  $[0 \ 1]$ : “response to low pain”

# Analysis of responses to multiple levels of painful stimuli: modelling



- Better approach: model as if two completely different stimuli
- Now, no pre-supposition about relationship between stimulation strength and response
- Can compare the size of the fits of the two regressors -
  - t-contrast  $[1 \ -1]$  : "is the response to high pain greater than that to low pain ?"
  - t-contrast  $[-1 \ 1]$  : "is the response to low pain greater than that to high pain ?"

# Analysis of responses to multiple levels of painful stimuli: modelling



- Better approach: model as if two completely different stimuli
- Now, no pre-supposition about relationship between stimulation strength and response
- Average response?
  - t-contrast  $[1 \ 1]$  : "is the average response to pain greater than zero?"

# Parametric Variation: Linear Trends

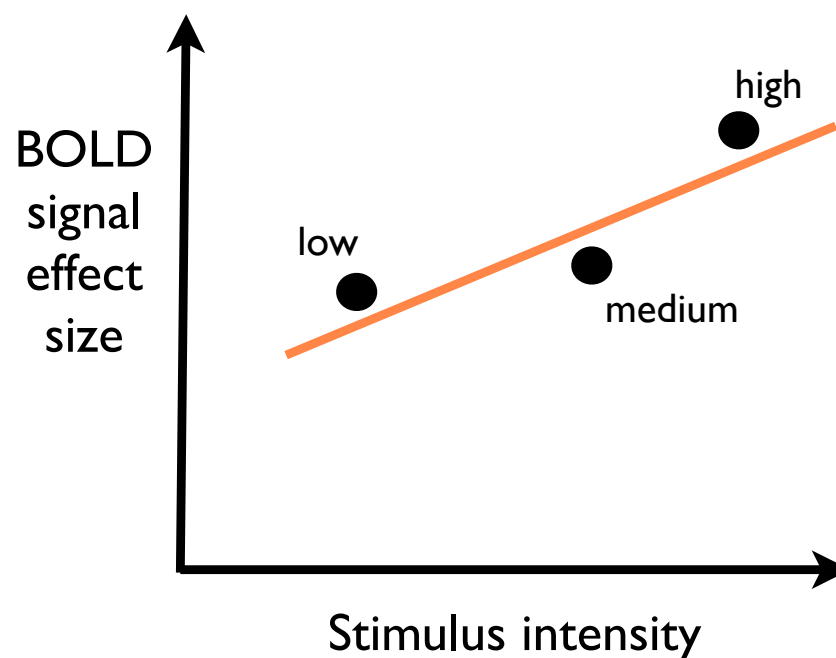


- Is there a linear trend between the BOLD response and stimulus intensity?

# Parametric Variation: Linear Trends

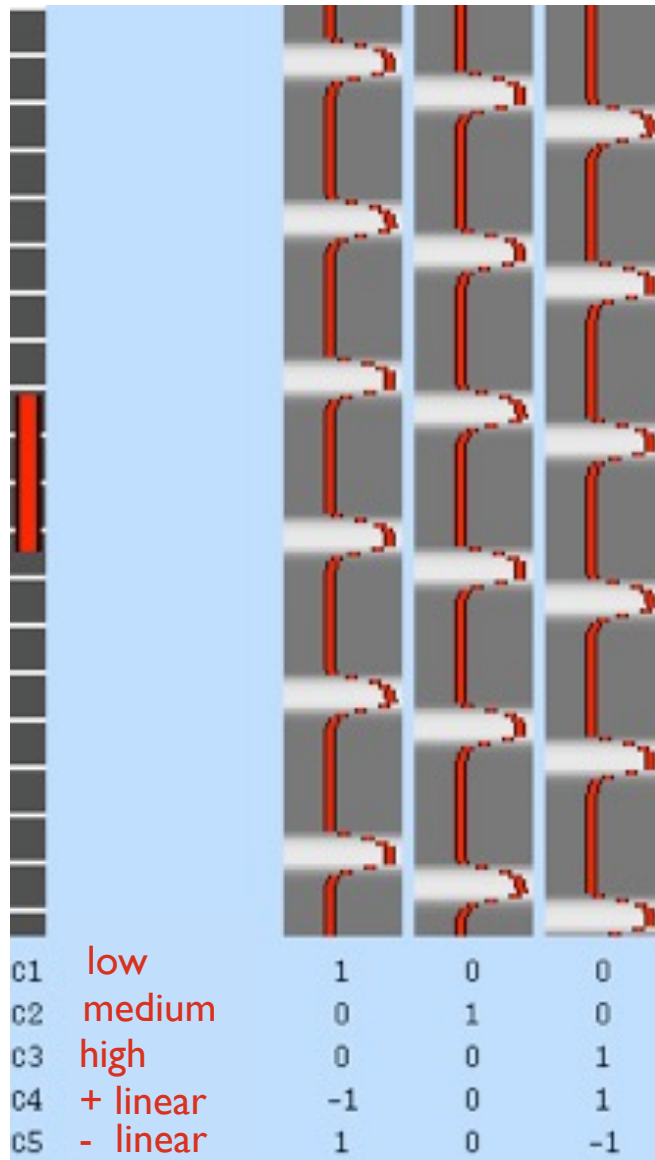


- Is there a linear trend between the BOLD response and stimulus intensity?



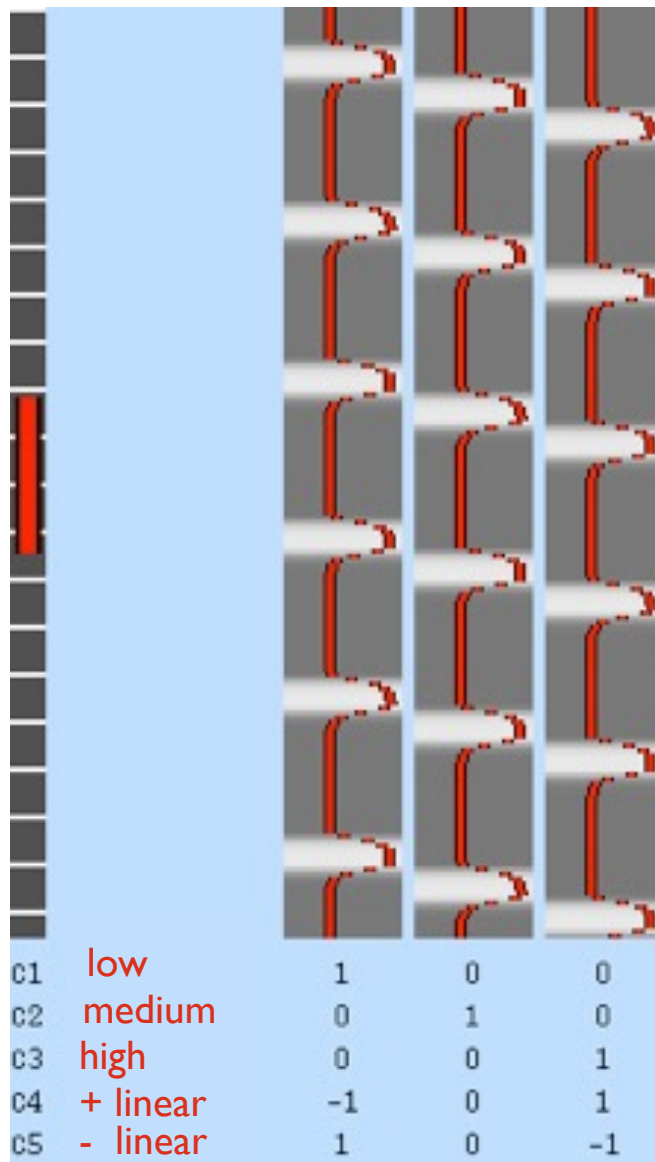


# Parametric Variation: Linear Trends

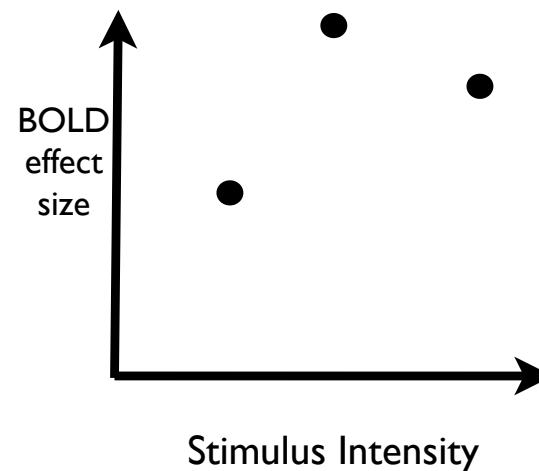


- A three-strength experiment
- Is there a linear trend between the BOLD response and some task variable?
- t-contrast  $[-1 \ 0 \ 1]$  : Linear trend

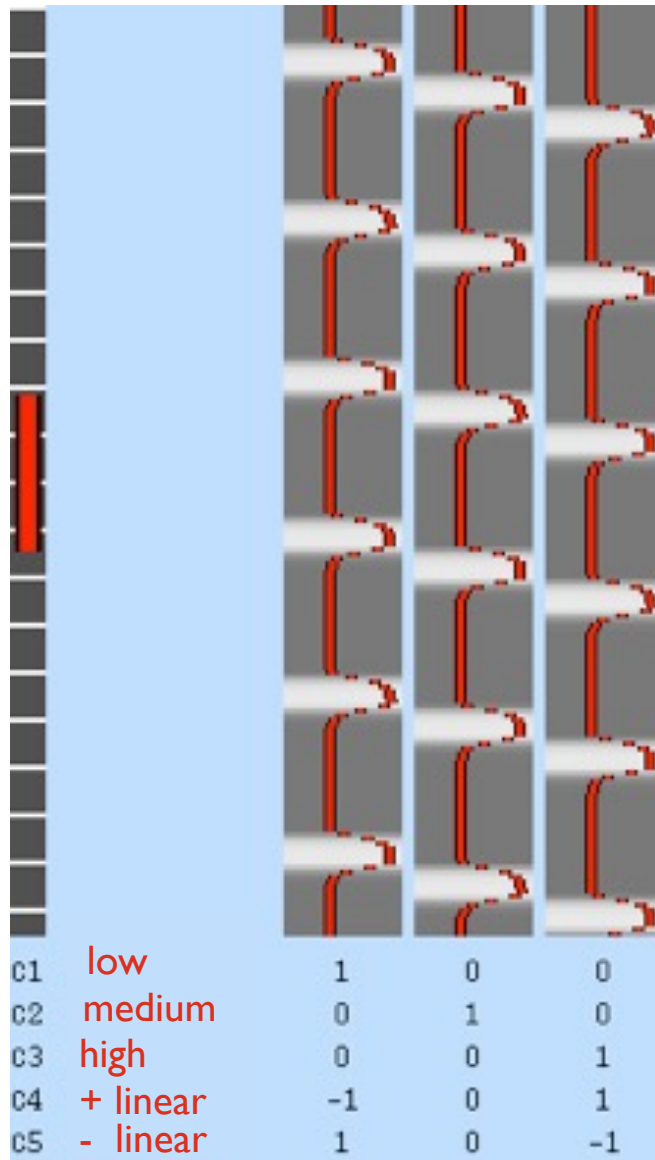
# Parametric Variation: Linear Trends



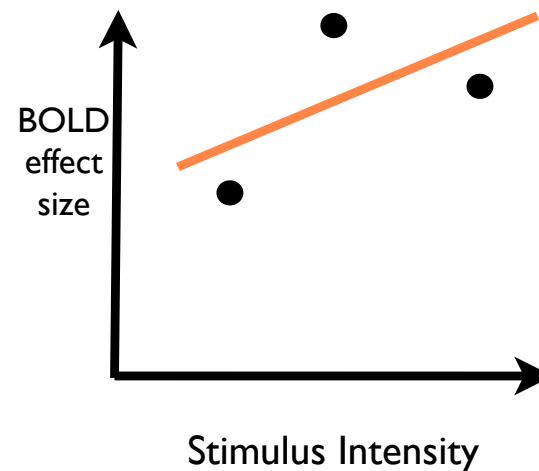
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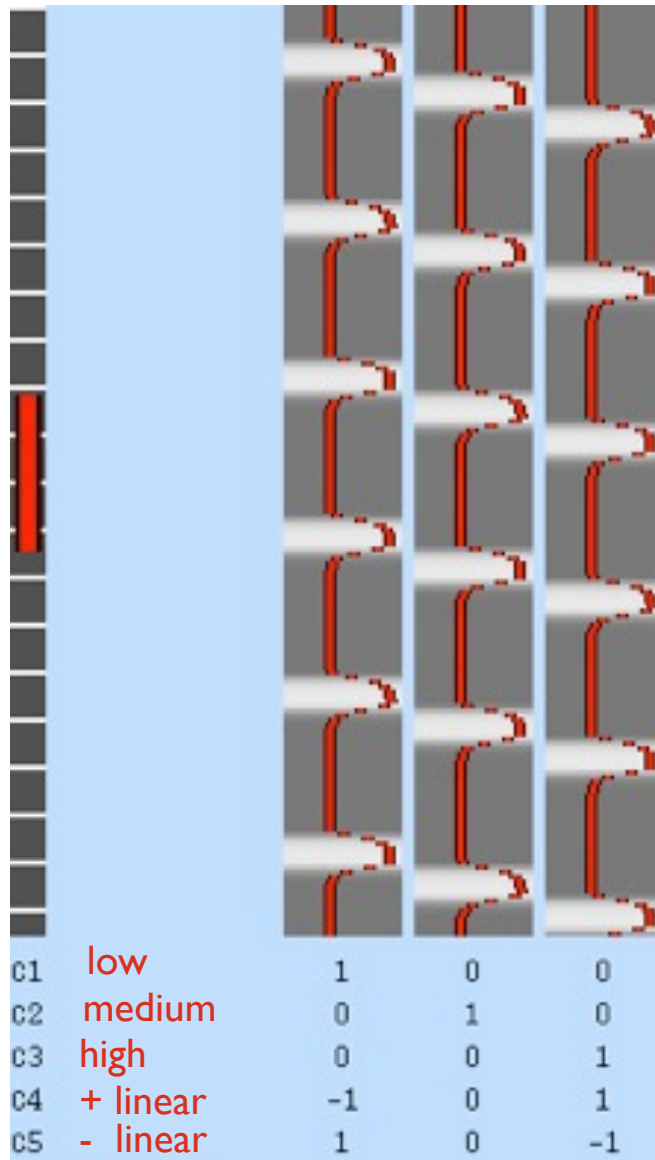
# Parametric Variation: Linear Trends



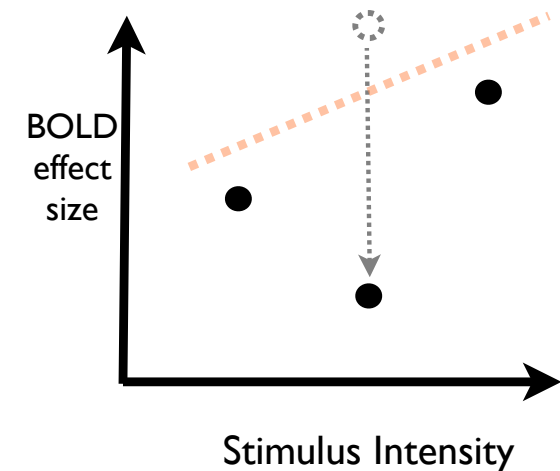
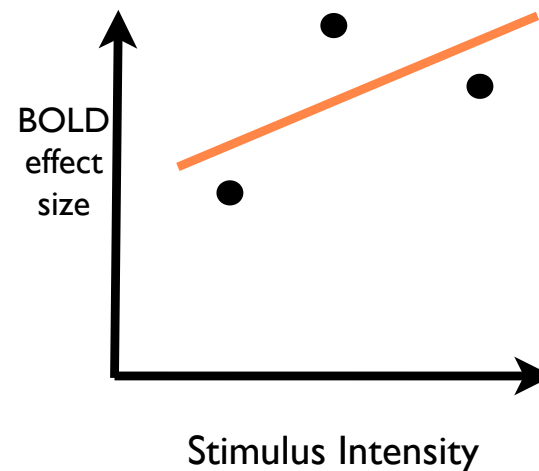
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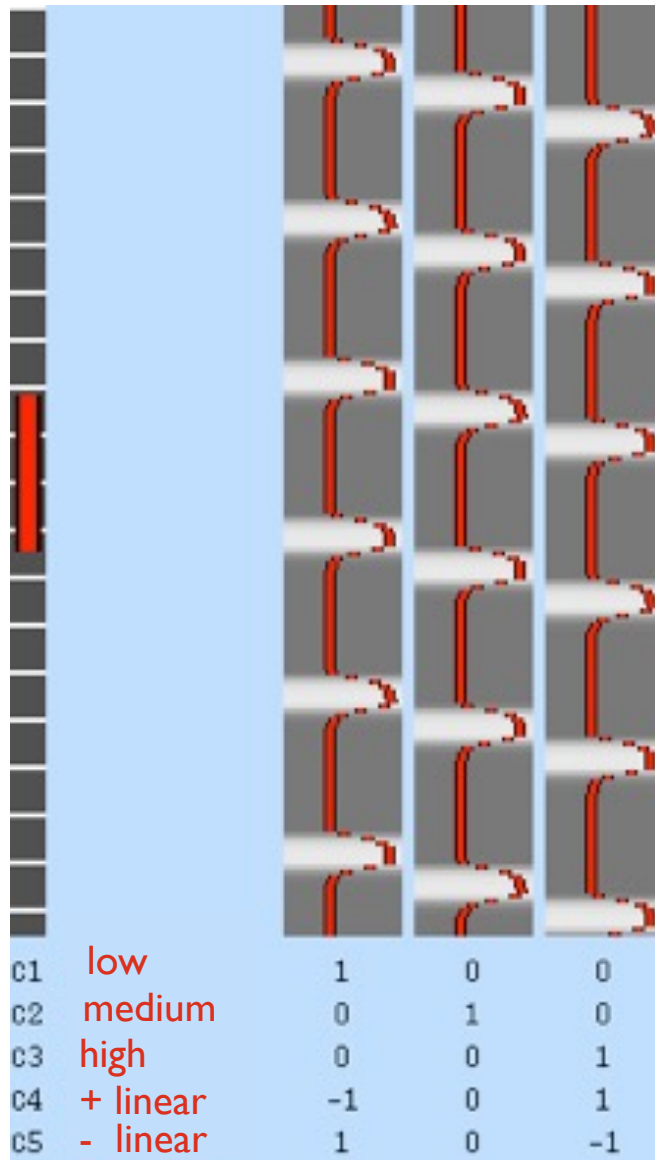
# Parametric Variation: Linear Trends



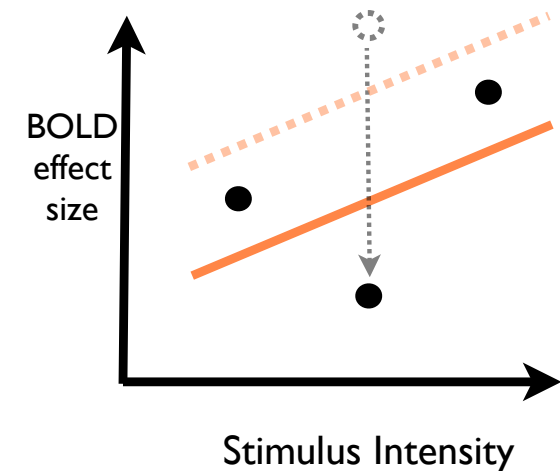
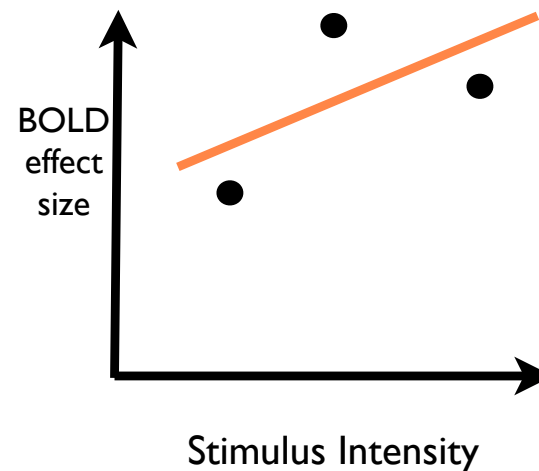
- A three-strength experiment
- Is there a linear trend between the BOLD response and some task variable?
- t-contrast  $[-1 \ 0 \ 1]$ : Linear trend



# Parametric Variation: Linear Trends

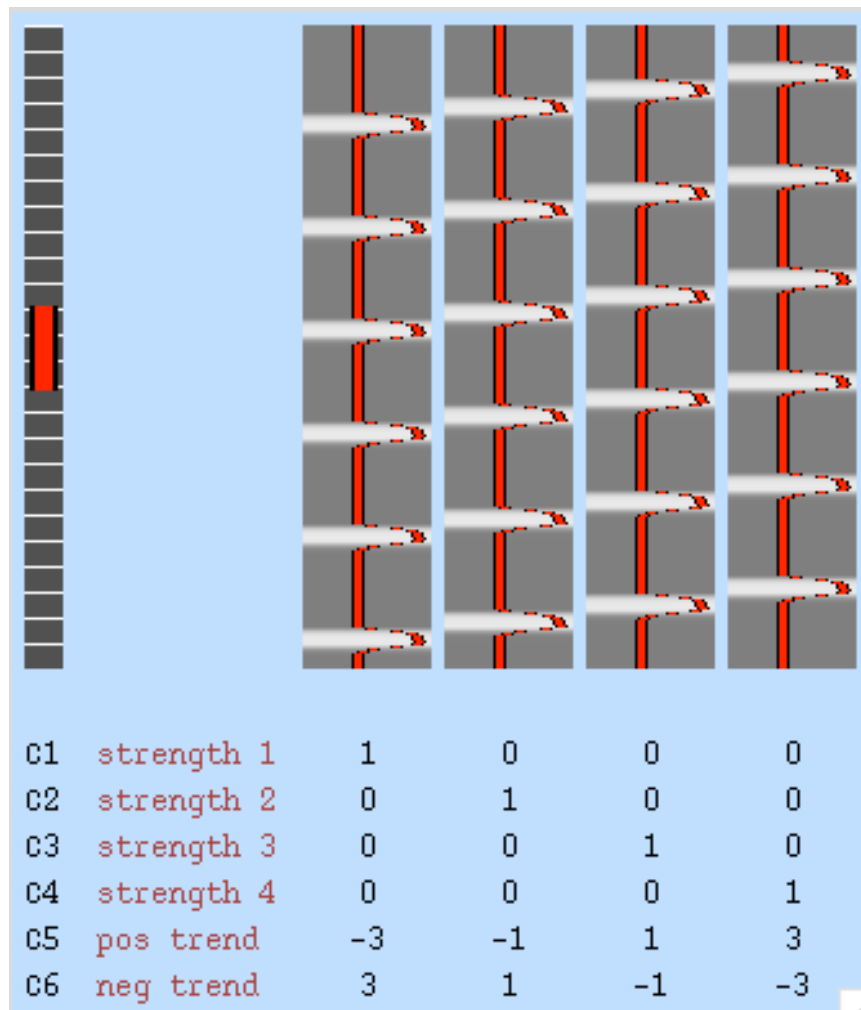


- A three-strength experiment
- Is there a linear trend between the BOLD response and some task variable?
- t-contrast  $[-1 \ 0 \ 1] : \text{Linear trend}$



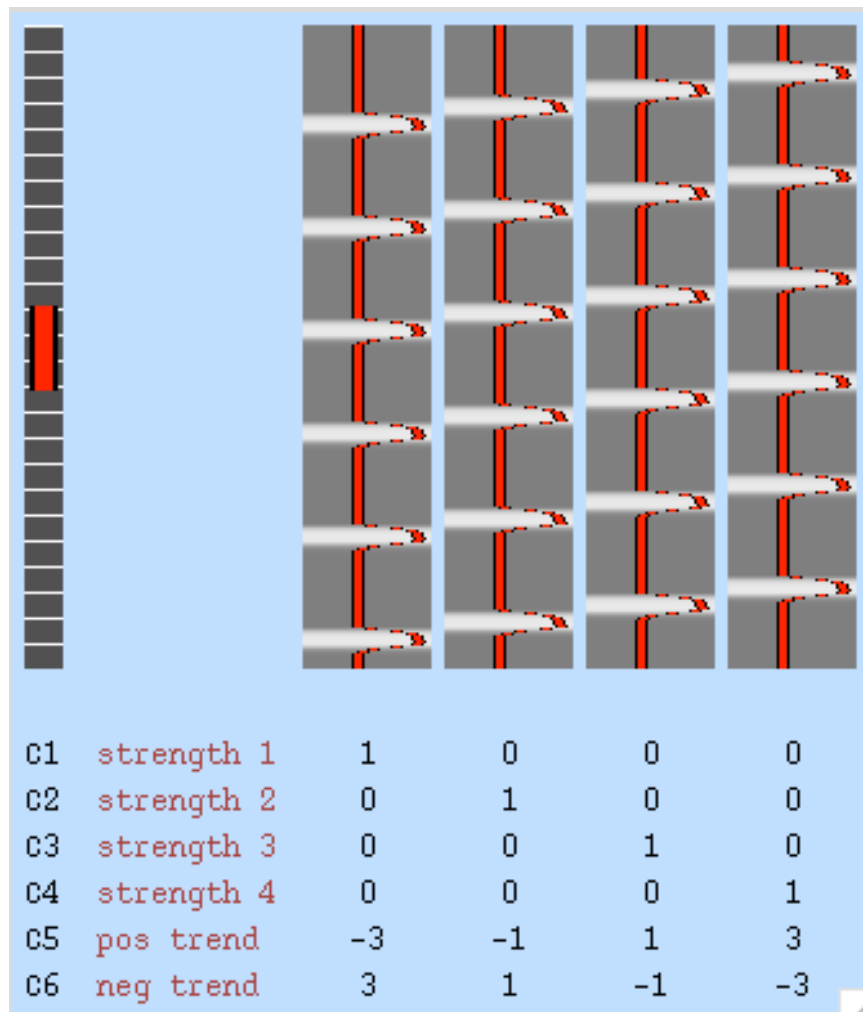
Slope  $(\beta_3 - \beta_1)$  is the same for both

# Parametric Variation: Linear Trends

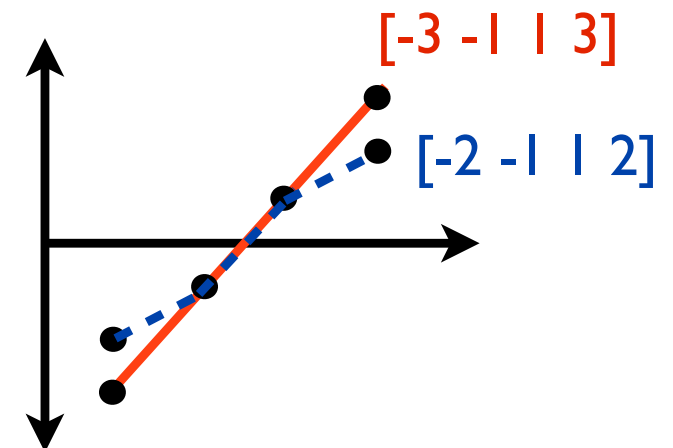


- A four-strength experiment
- t-contrast  $[-3 \ -1 \ 1 \ 3]$  :  
Positive linear trend

# Parametric Variation: Linear Trends



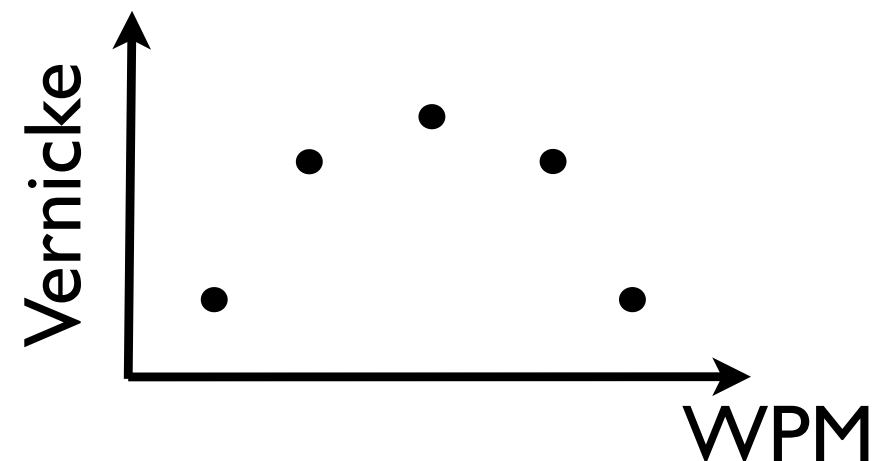
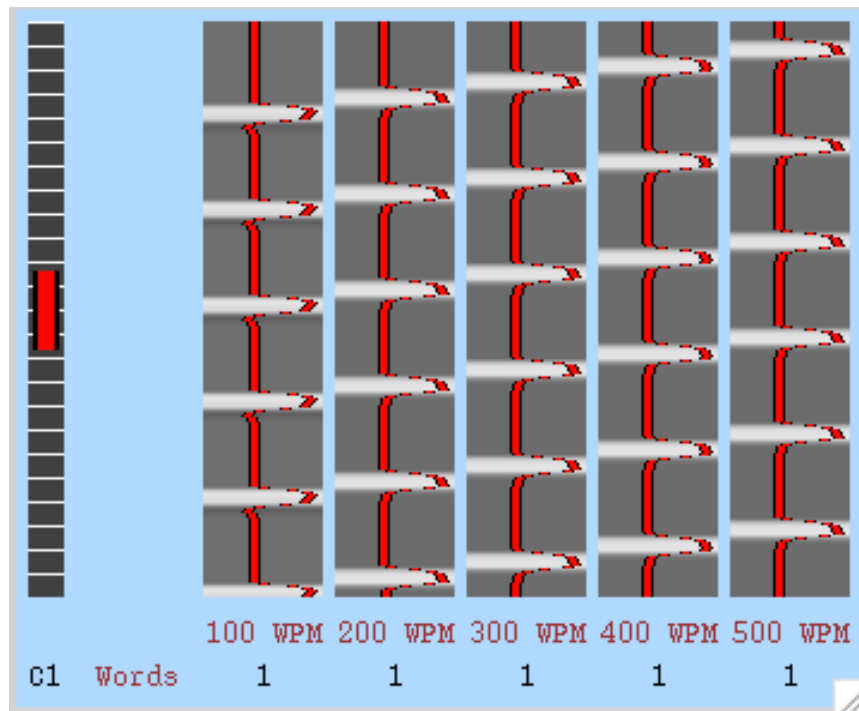
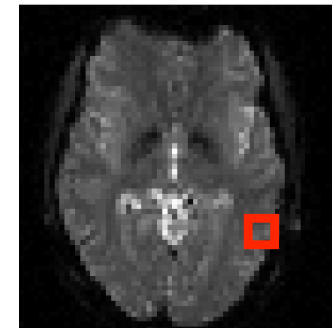
- A four-strength experiment
- t-contrast  $[-3 \ -1 \ 1 \ 3]$  :  
Positive linear trend



# But what if it isn't that predictable?



Auditory word presentation  
at different rates

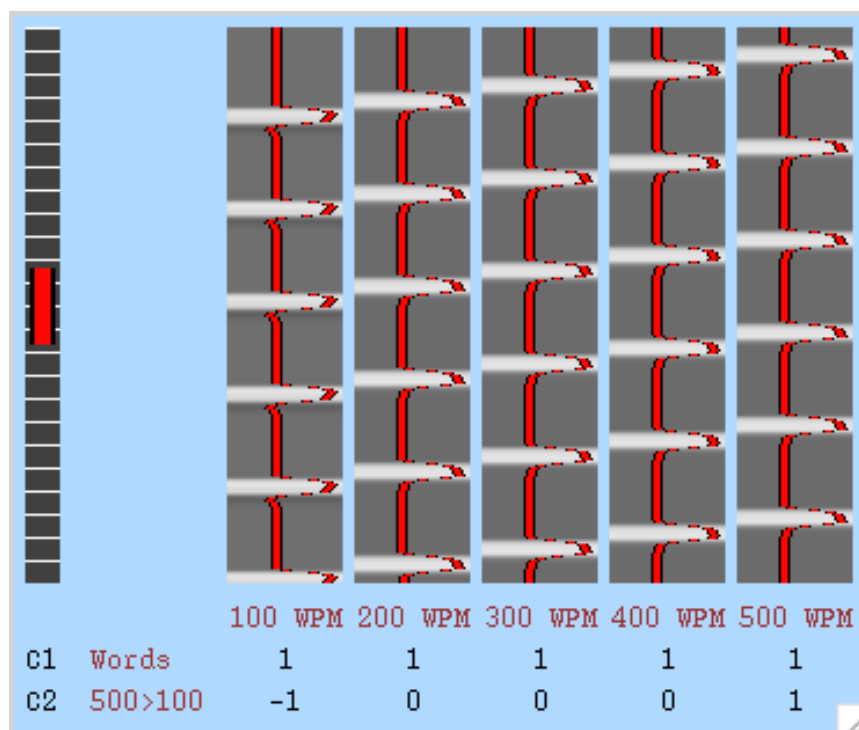




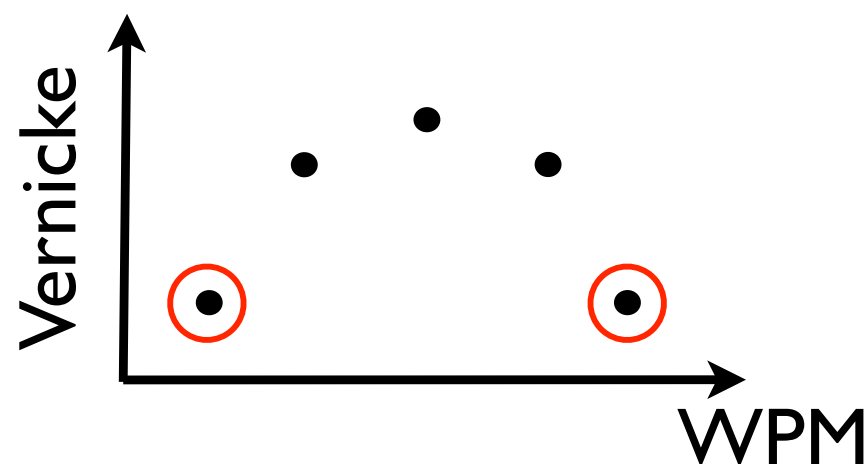
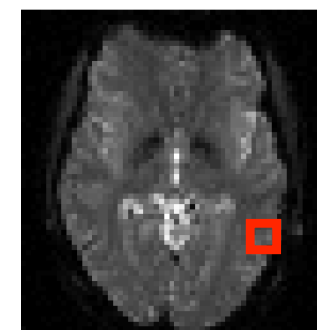
# But what if it isn't that predictable?



Given this design what would be “reasonable” questions to ask?



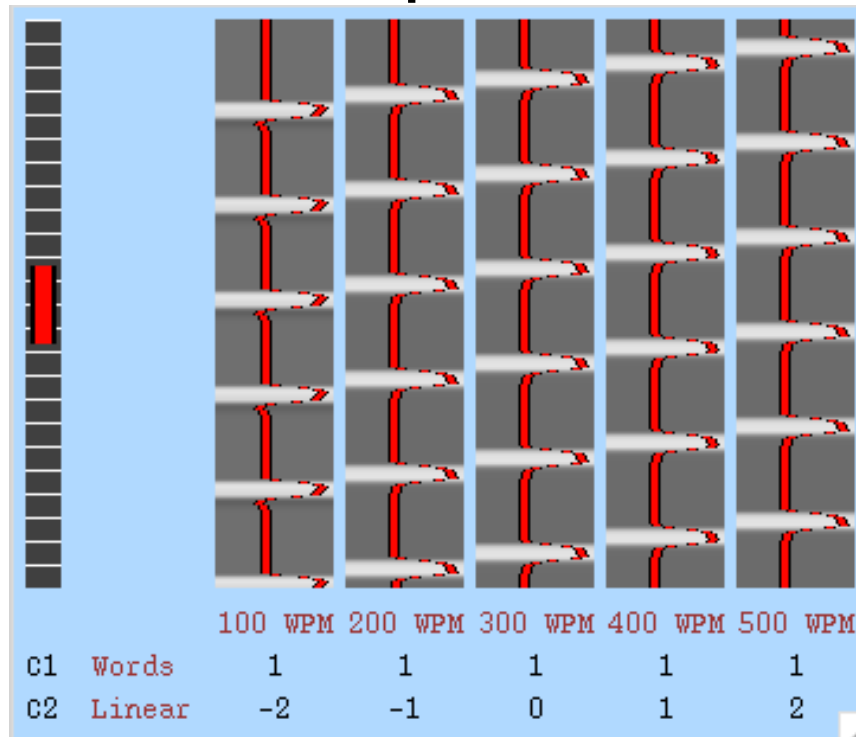
More activation to 500 than to 100 WPM?



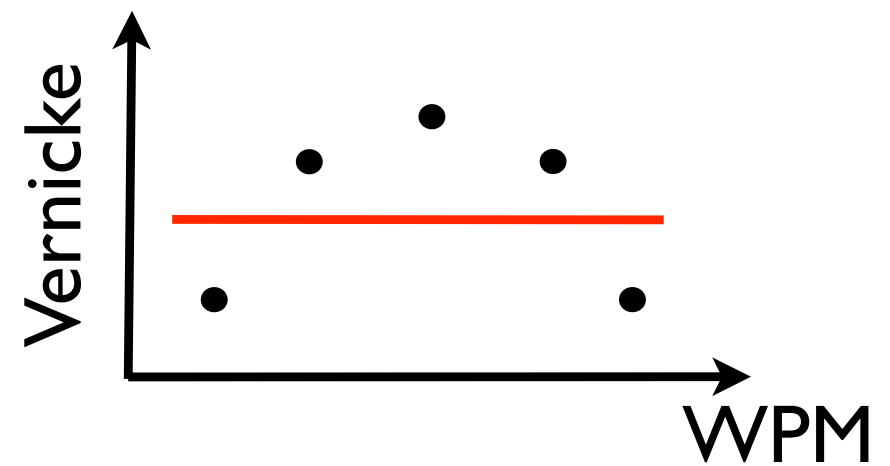
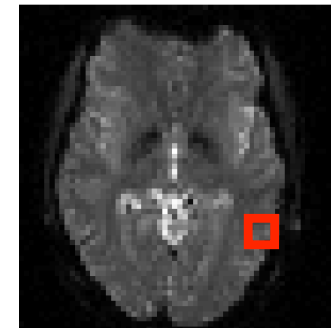
# But what if it isn't that predictable?



Given this design what would be “reasonable” questions to ask?



Activation proportional to WPM?

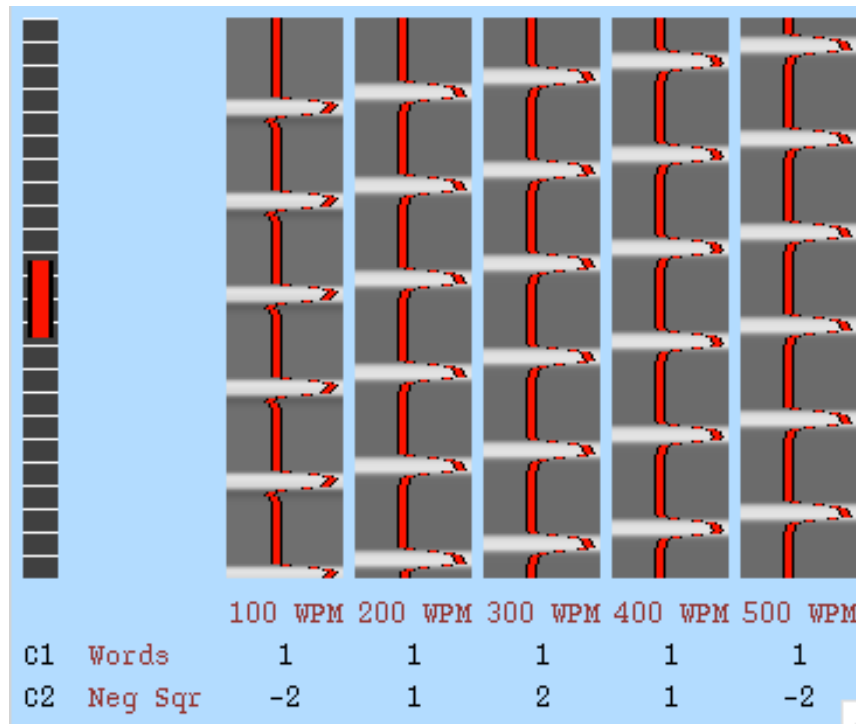


Still no...

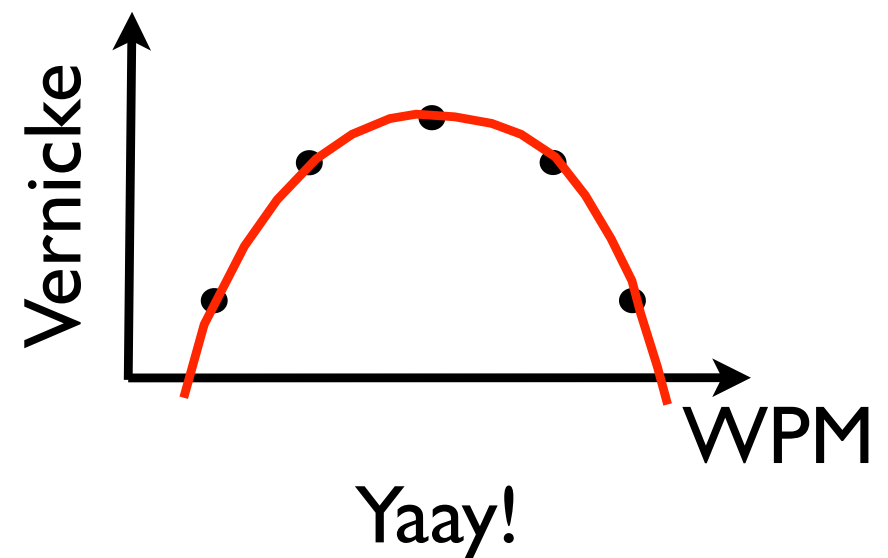
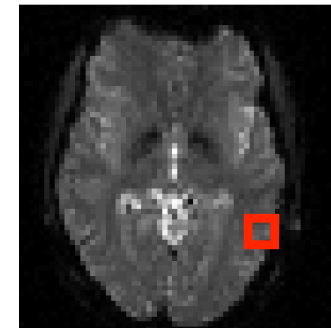
# But what if it isn't that predictable?



Given this design what would be “reasonable” questions to ask?



Inversely proportional to WPM squared?

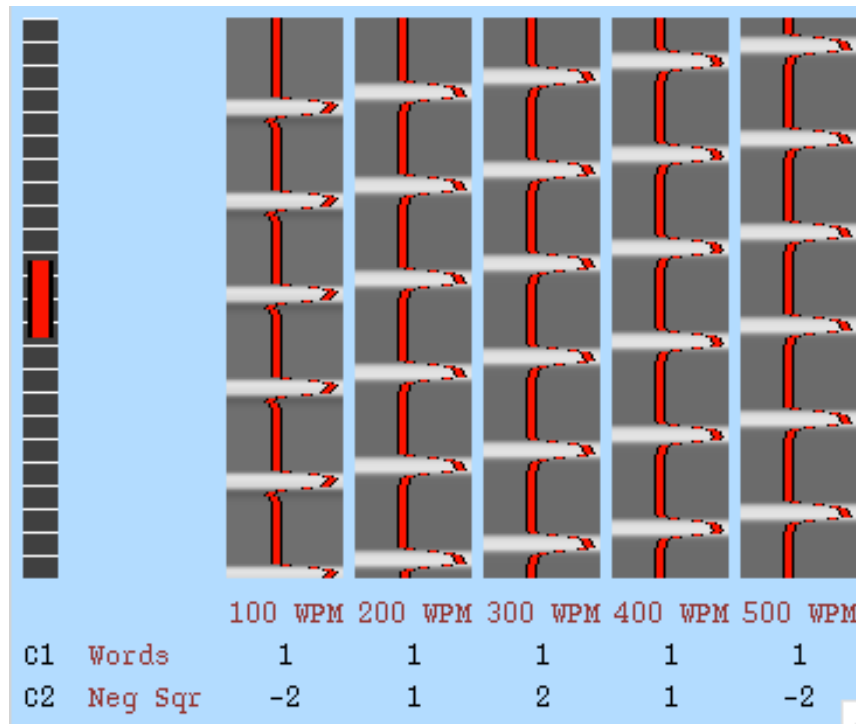


True story

# But what if it isn't that predictable?

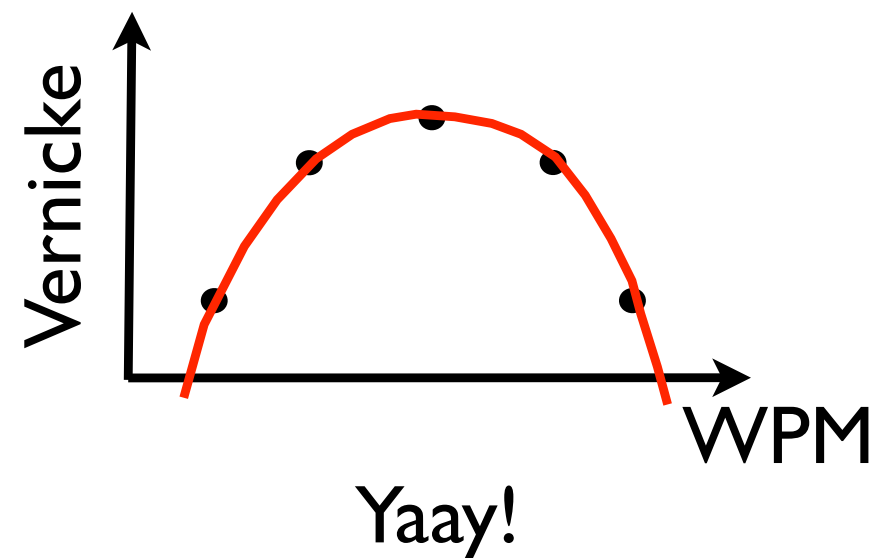


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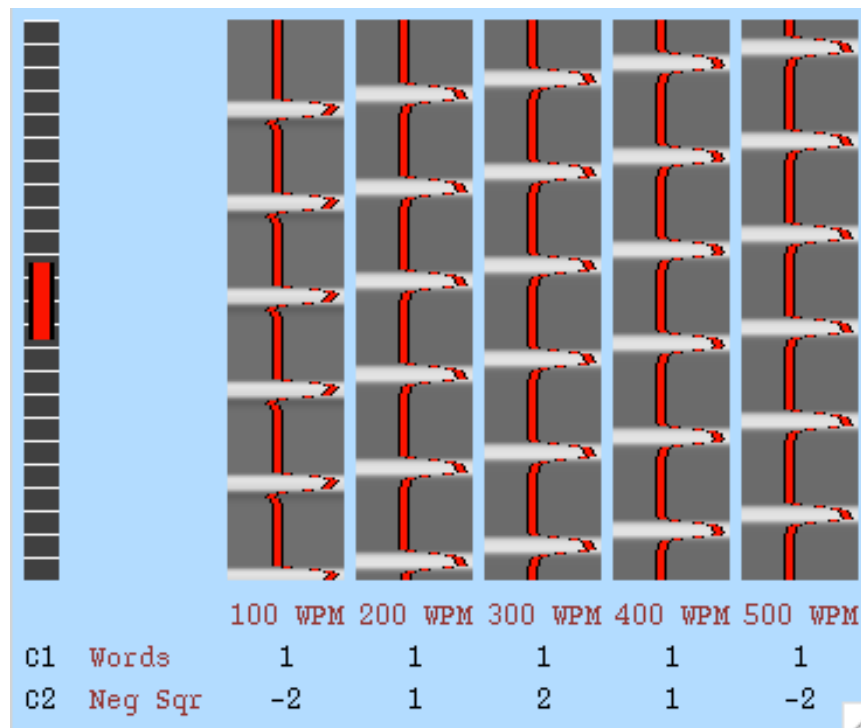


Inversely proportional to WPM squared?

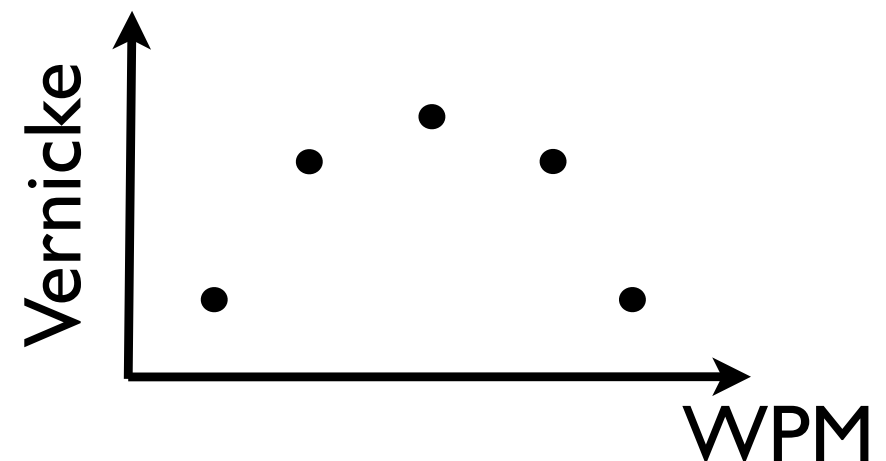
But seriously ... would you have asked that question?



# But what if it isn't that predictable?

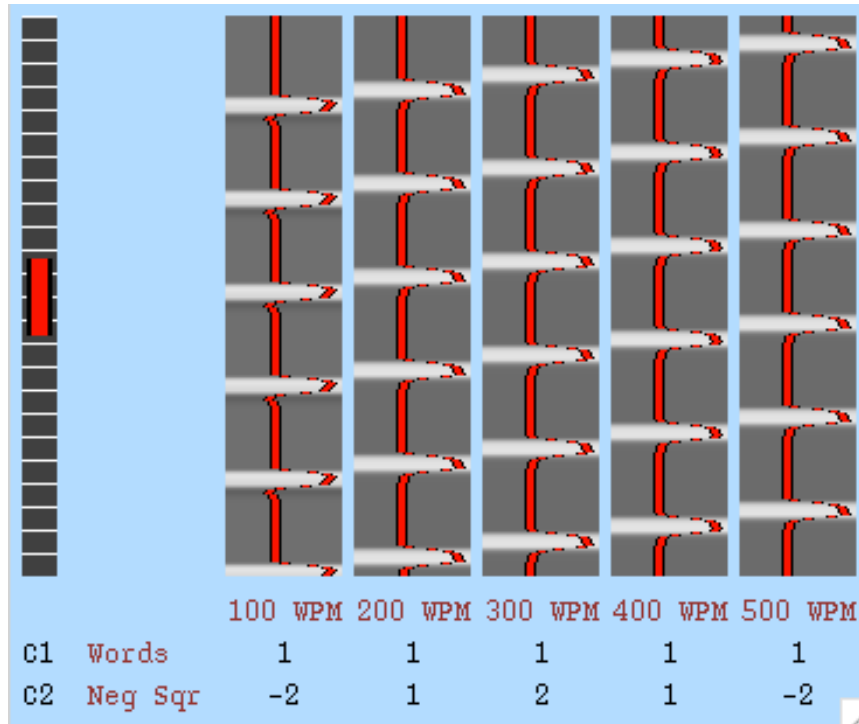


There is a (very real) risk of missing interesting but unpredicted responses



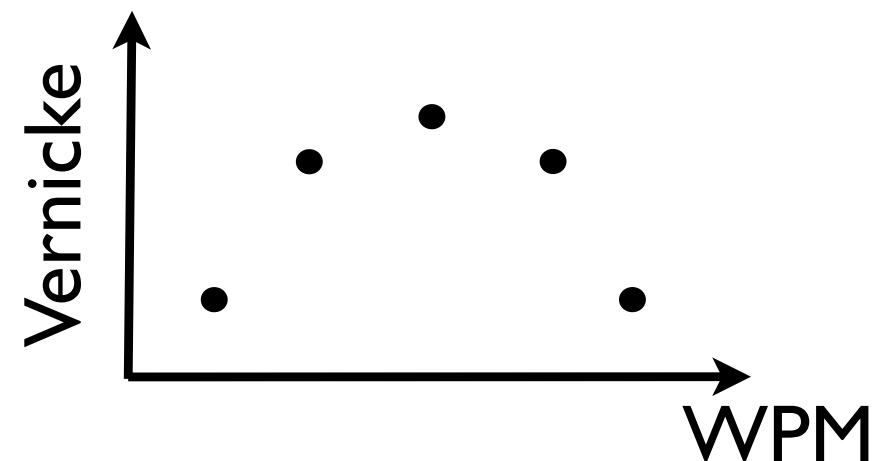
What can we do about that?

# F-contrasts to the rescue

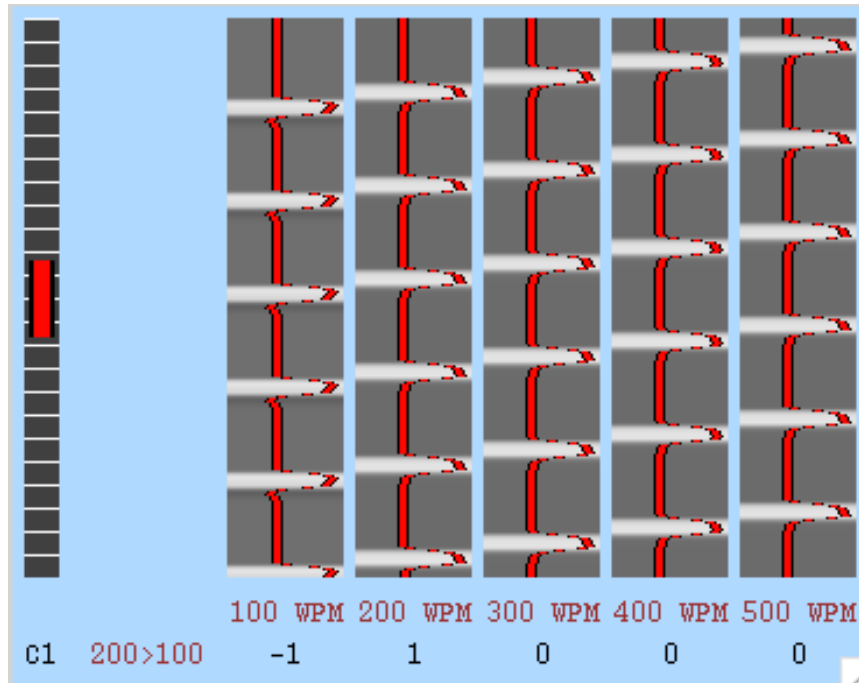


We can define an F-contrast that spans “the range of possible responses”

An F-contrast is a series of questions (*t*-contrasts) with an OR between them

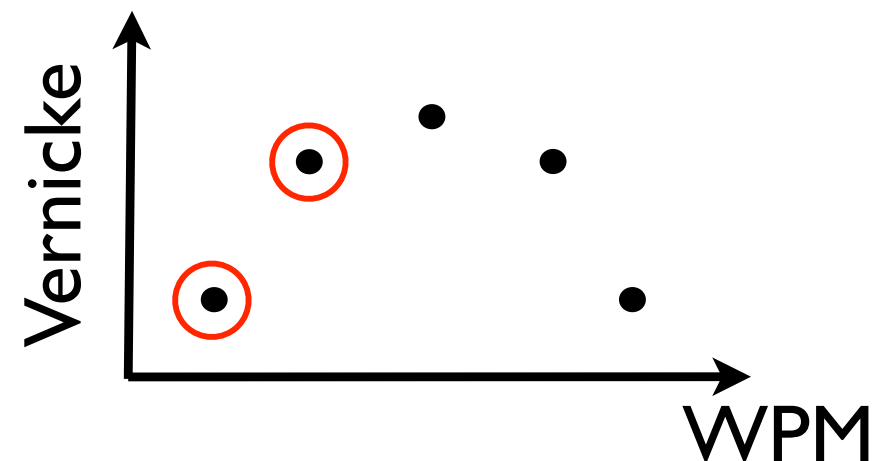


# F-contrasts to the rescue

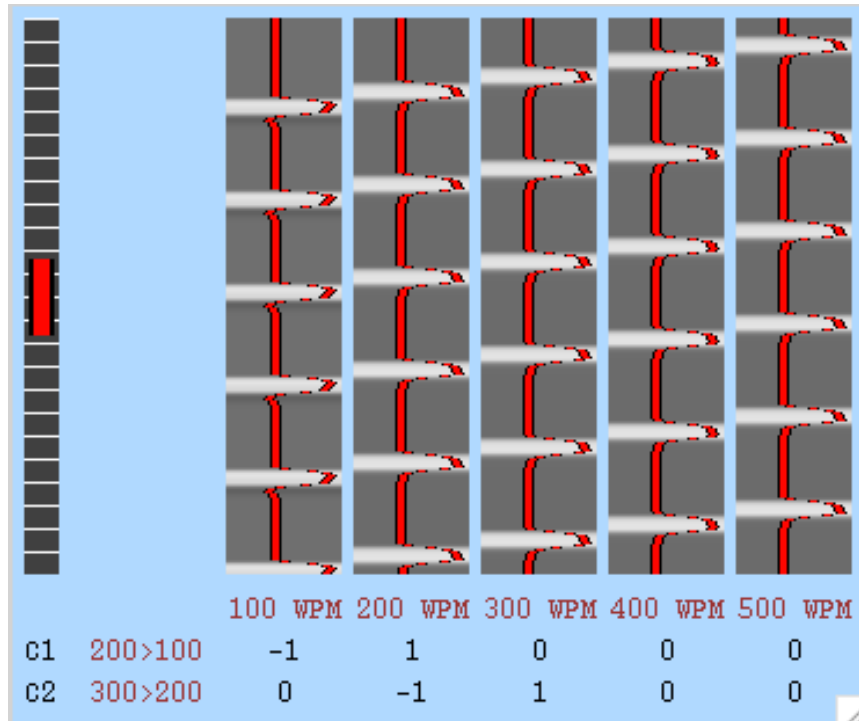


Let's start with "Greater activation to 200 than 100 WPM"

We can define an F-contrast that spans "the range of possible responses"



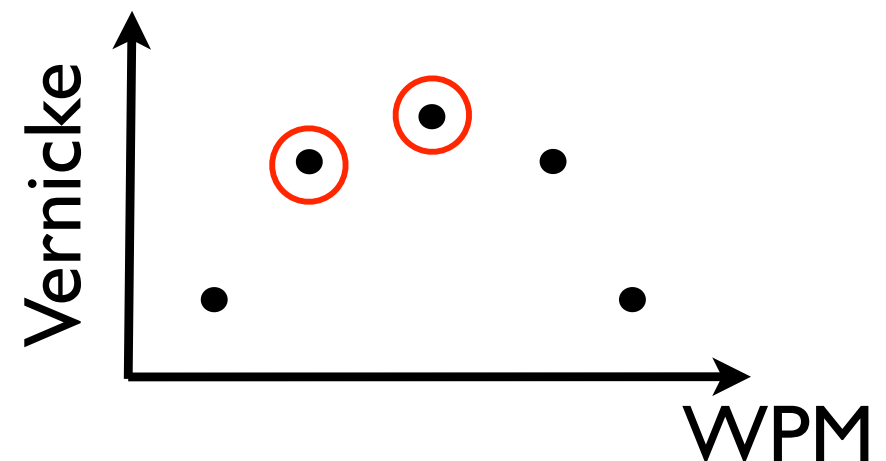
# F-contrasts to the rescue



OR

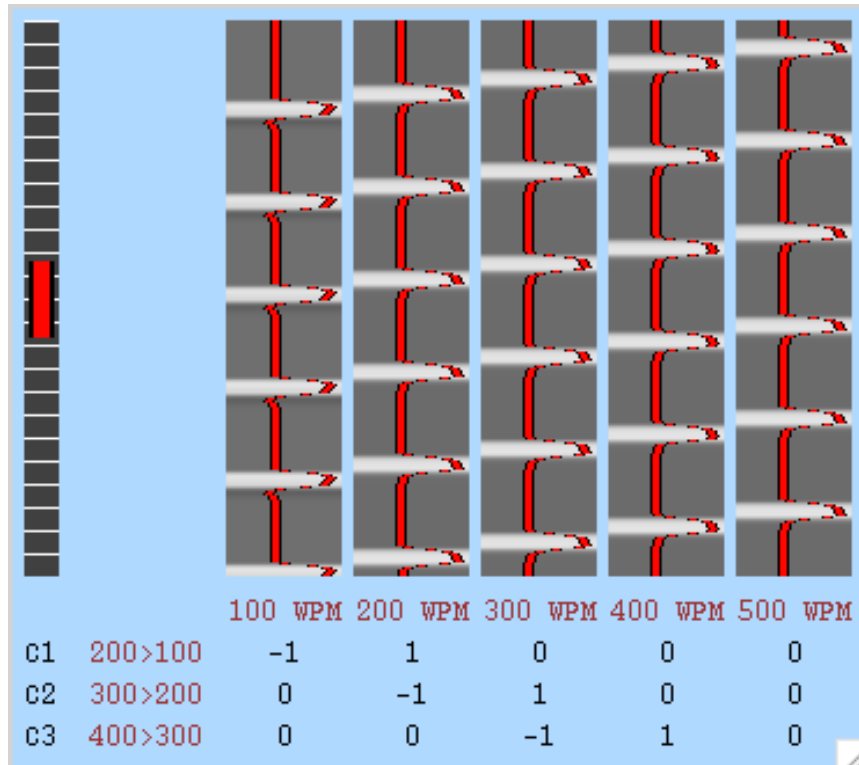
300WPM > 200WPM

We can define an F-contrast that spans “the range of possible responses”





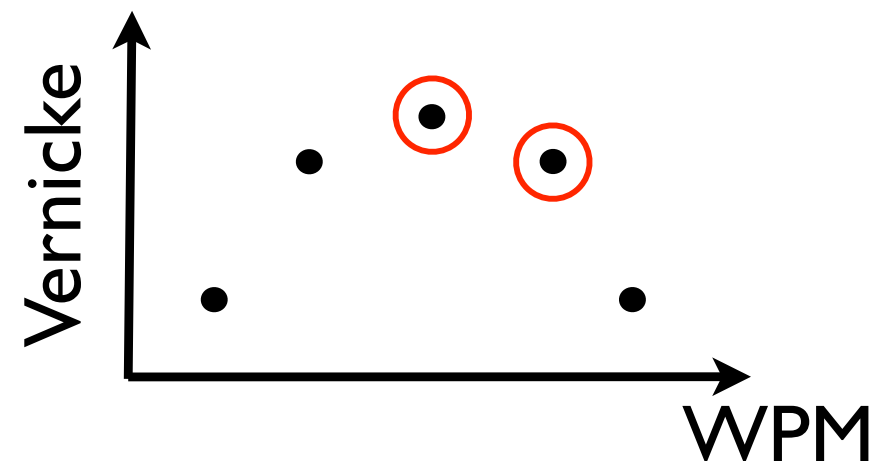
# F-contrasts to the rescue



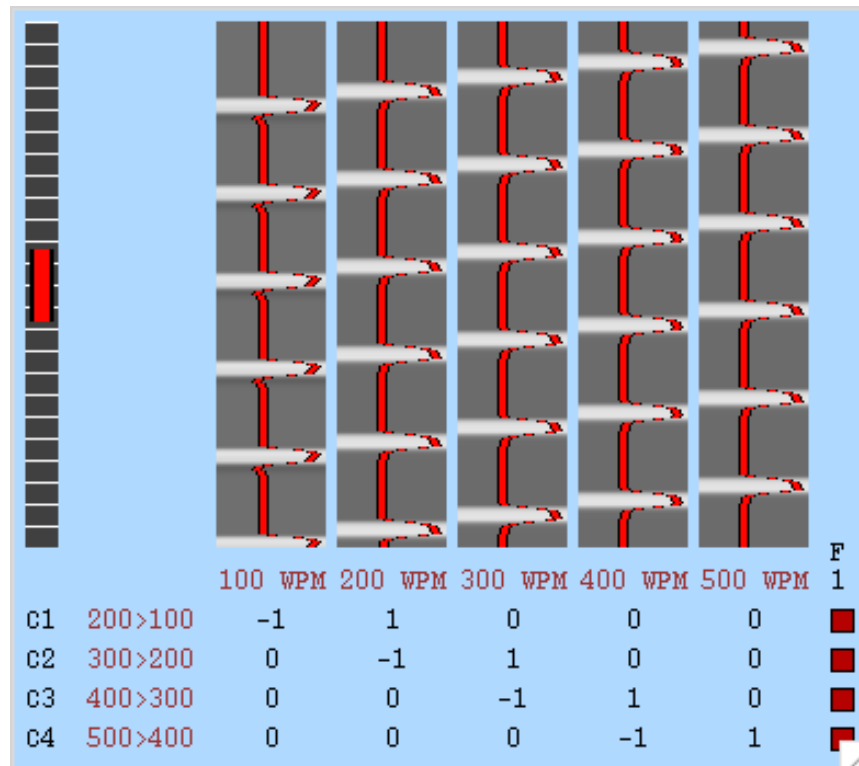
OR

400WPM > 300WPM

We can define an F-contrast that spans “the range of possible responses”



# F-contrasts to the rescue



EVs

Contrasts & F-tests

Setup contrasts & F-tests for

Original EVs

Contrasts

4

F-tests

1

Paste

Title

EV1

EV2

EV3

EV4

EV5

F1

OC1

200>100

-1

1

0

0

0

OC2

300>200

0

-1

1

0

0

OC3

400>300

0

0

-1

1

0

OC4

500>400

0

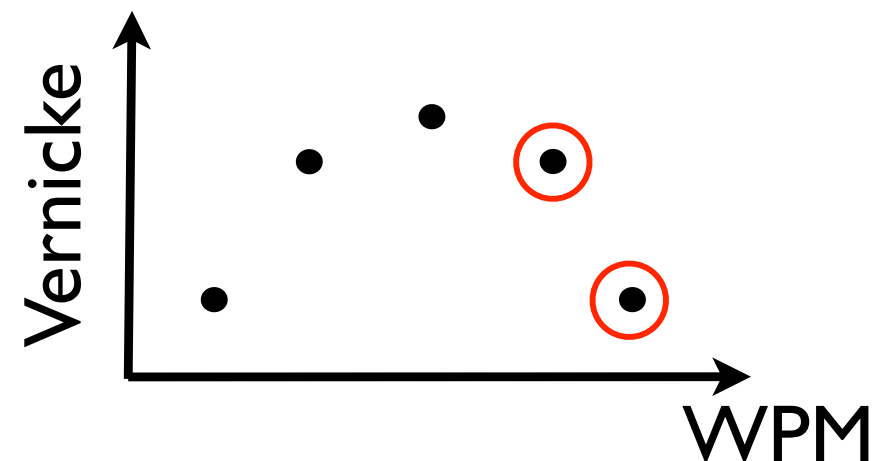
0

0

-1

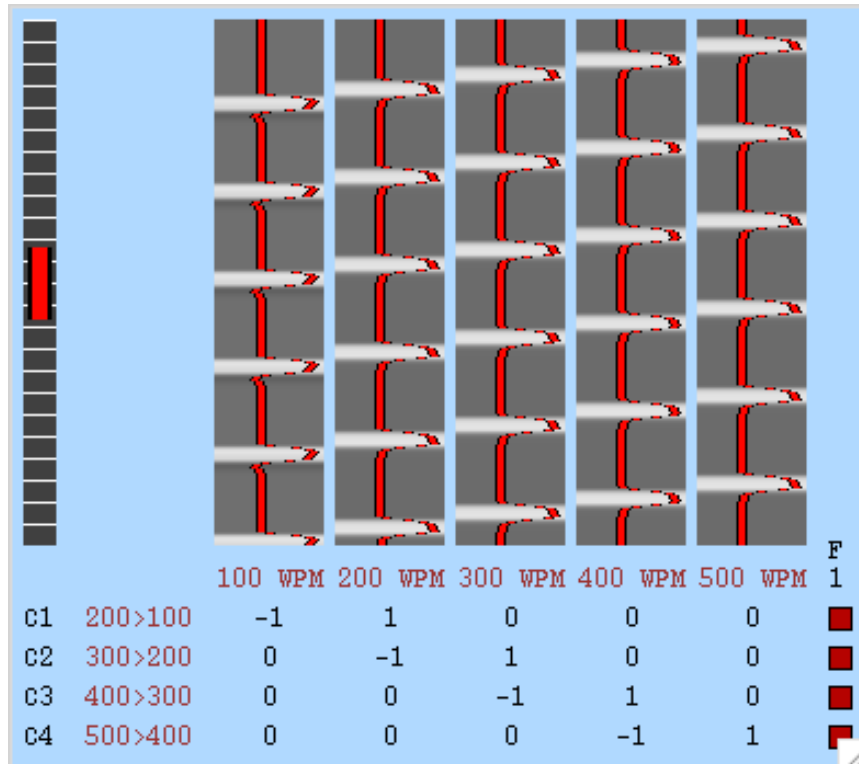
1

N.B.

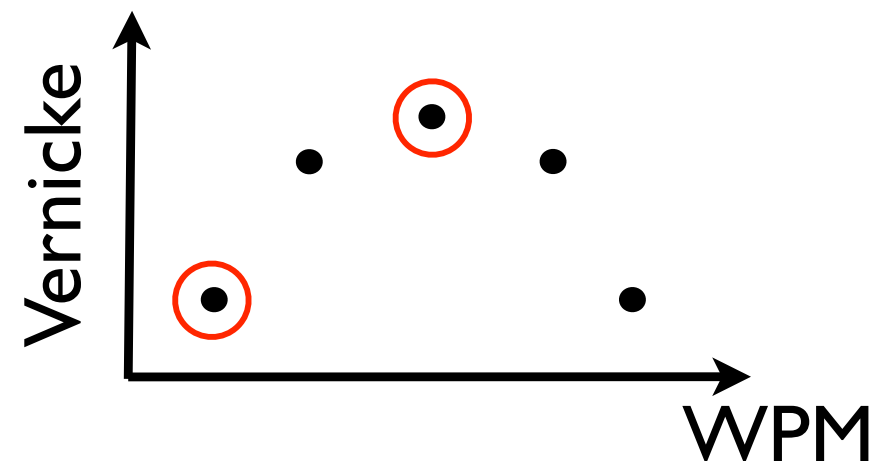


OR  
500WPM > 400WPM

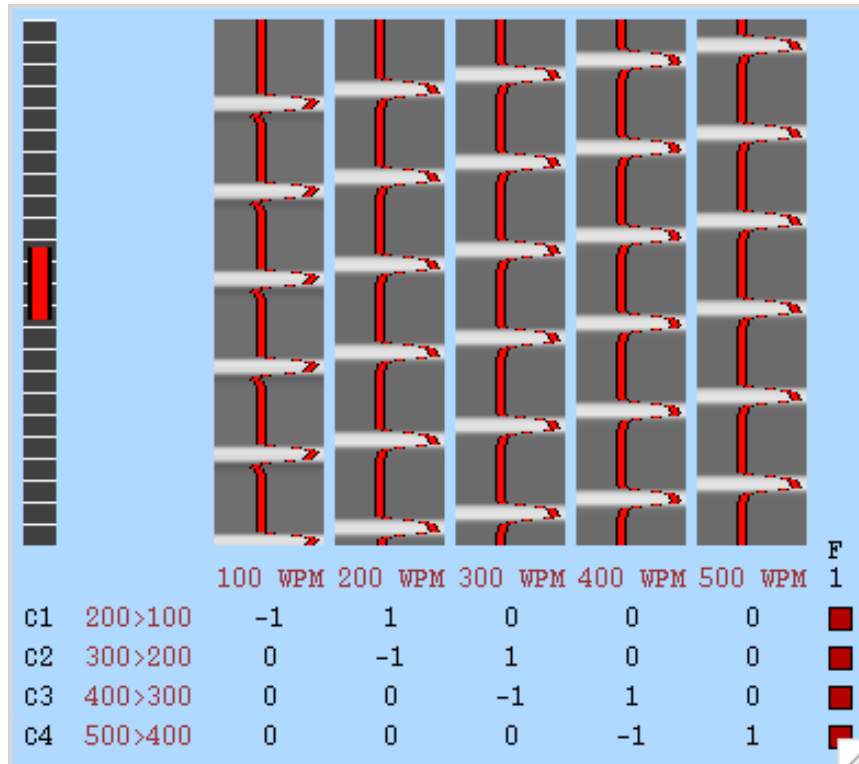
# F-contrasts to the rescue



But ... that doesn't span all possible response, what about for example  $300 > 100$ ?

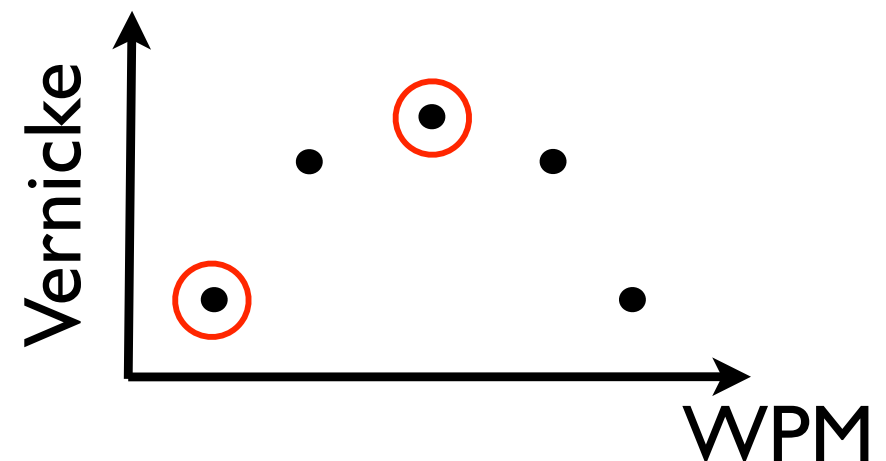


# F-contrasts to the rescue

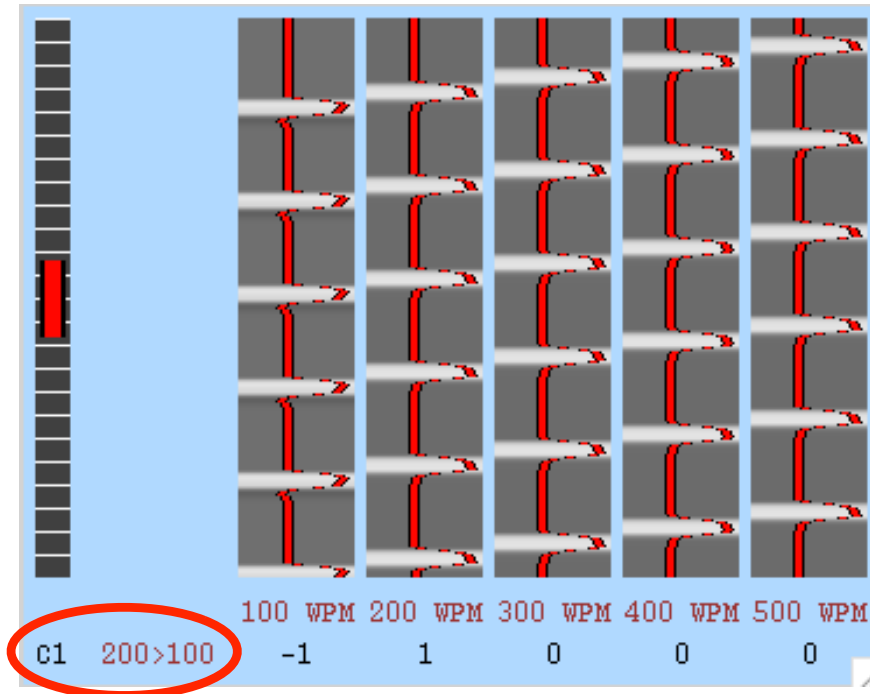


300>100 implies  
200>100 AND/OR 300>200  
which we have covered

But ... that doesn't span all possible response, what about for example 300>100?



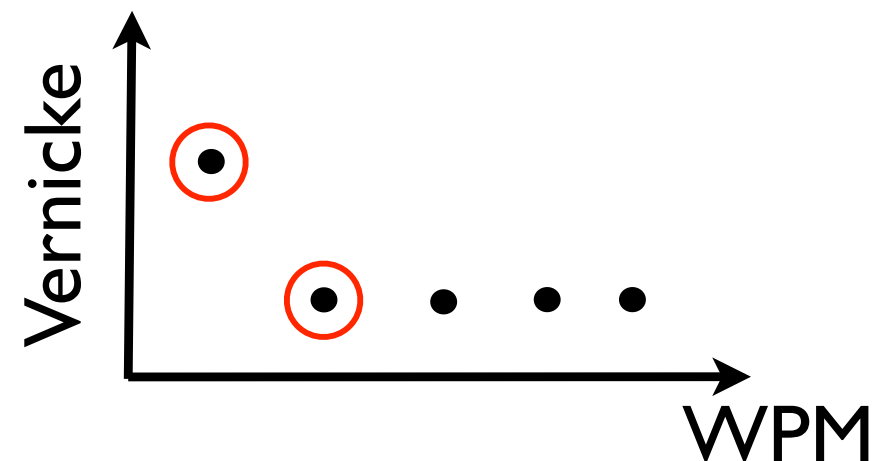
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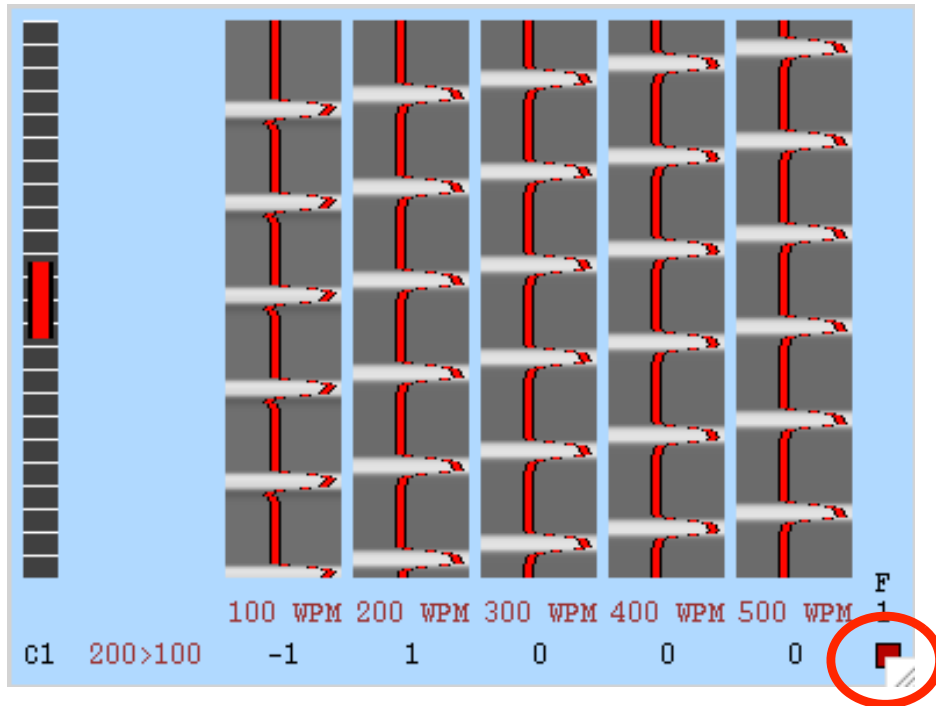
This *t*-contrast asks  
“where is 200>100?”

F-contrasts are  
bi-directional

But ... what about for  
example 100>200, you  
haven't covered that?



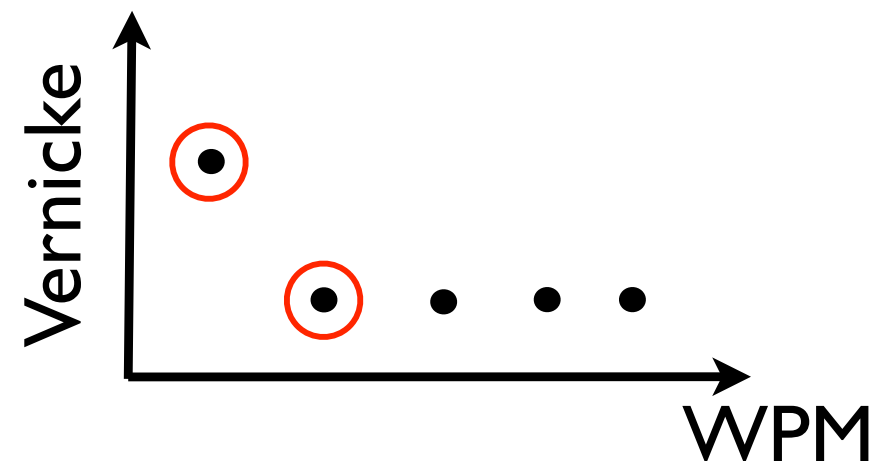
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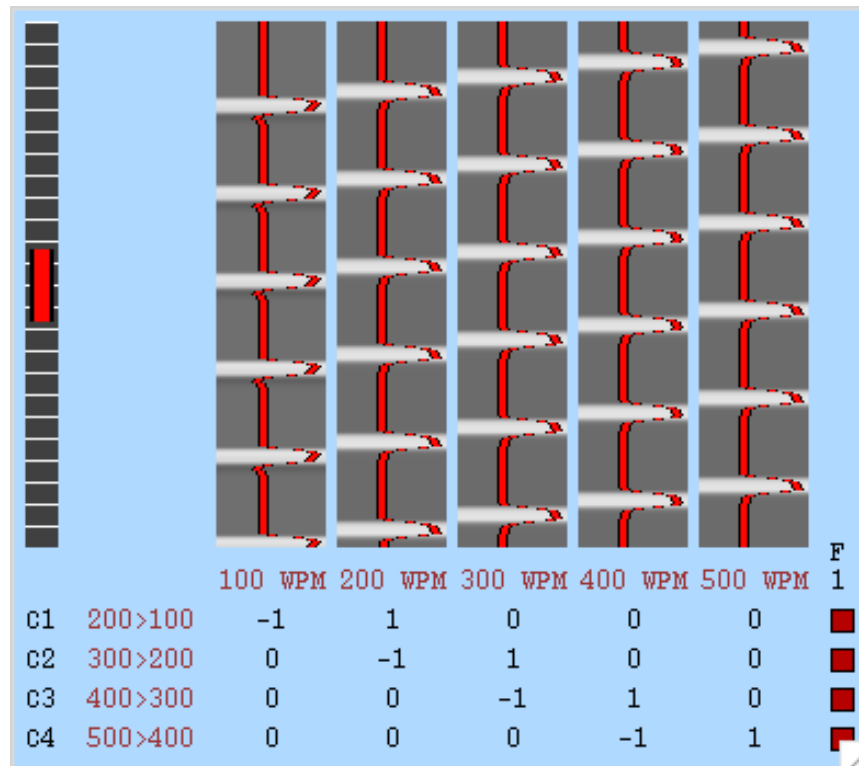
But this F-contrast asks  
“where is  $200 \neq 100$ ?”

F-contrasts are  
bi-directional

But ... what about for  
example  $100 > 200$ , you  
haven't covered that?

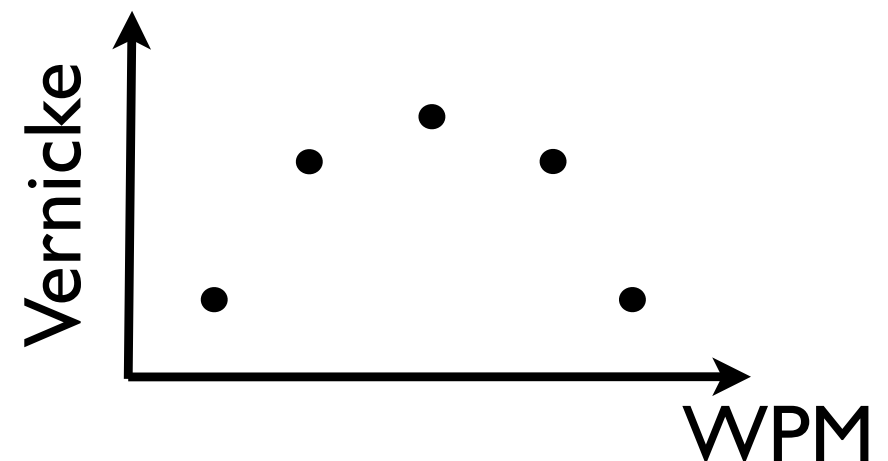


# F-contrasts to the rescue



EVs		Contrasts & F-tests									
		Setup contrasts & F-tests for <span>Original EVs</span>									
		Contrasts	4	F-tests	1						
		Paste	Title		EV1	EV2	EV3	EV4	EV5	F1	
	OC1		200>100		-1	1	0	0	0		
	OC2		300>200		0	-1	1	0	0		
	OC3		400>300		0	0	-1	1	0		
	OC4		500>400		0	0	0	-1	1		

N.B.





# Advanced Analysis: Parametric Designs

## Summary:

- Important to have separate EVs (and parameters) per level of stimulus, otherwise assuming an exact linear response
- Linear trends require contrasts that are centred about zero and with even intervals
- Going beyond linear trends can be done with F-tests to look for arbitrary response shapes



# Advanced Analysis: Factorial Designs and Interactions



Scenario:

Investigating in multi-sensory regions

Specific questions:

What regions show responses to vision, touch

What regions respond significantly to both?

Are responses additive where there is both visual and touch stimulation, or is there an interaction?

Solution:

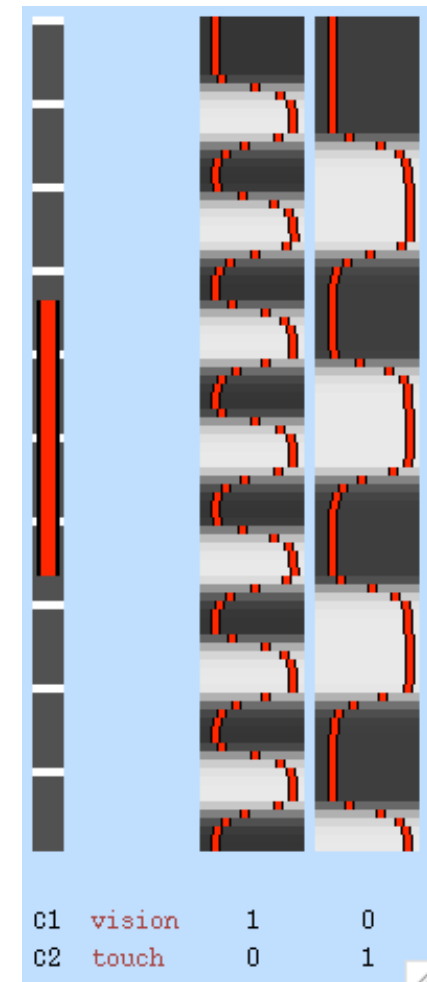
Specific regressors

Contrast masking

# Multisensory study



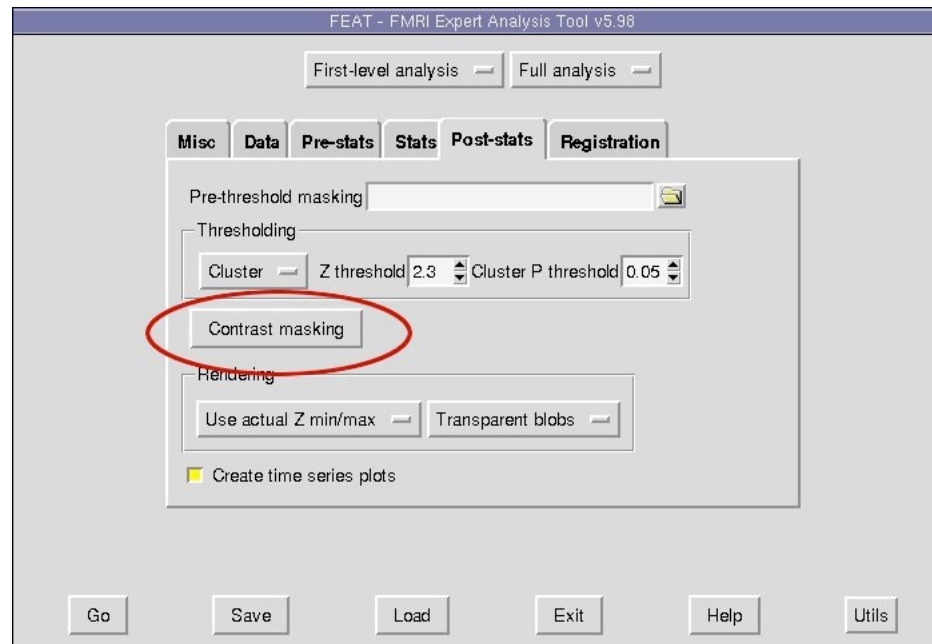
- EV1 models vision on/off
- EV2 models touch on/off
- Can generate simple contrasts for:
- vision activation/deactivation [ 1 0 ]
- touch activation/deactivation [ 0 1 ]
- differences in responses [ 1 -1 ]
- Regions showing both visual and tactile response??
- Not [ 1 1 ]: this only assesses the average



# Contrast Masking



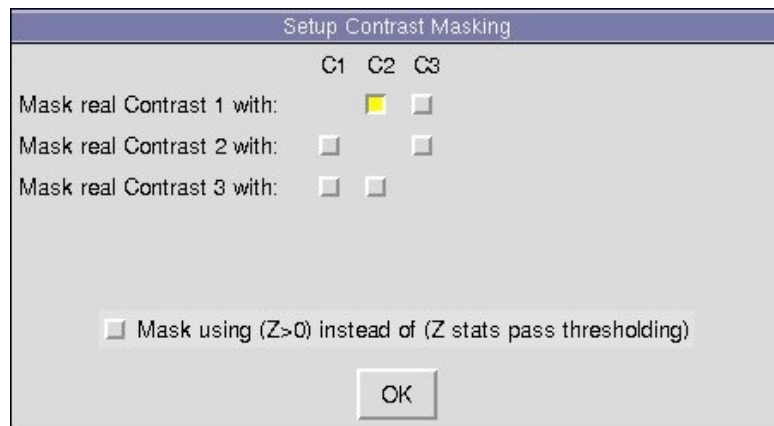
- Often it is of interest to identify regions showing significant effects in multiple contrasts (e.g. responds to visual AND tactile stimulations)
- This can be achieved by masking a thresholded z image for a chosen contrast using the thresholded z image from one or more other contrasts.



# Contrast Masking



- Often it is of interest to identify regions showing significant effects in multiple contrasts (e.g. responds to visual AND tactile stimulations)
- This can be achieved by masking a thresholded z image for a chosen contrast using the thresholded z image from one or more other contrasts.

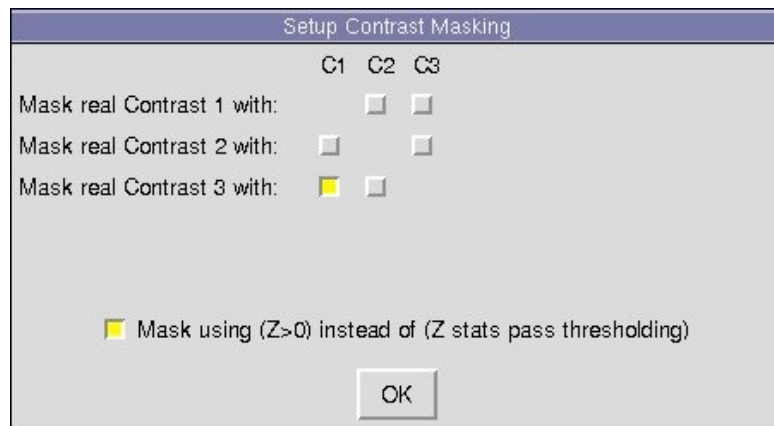


For example, say we had two t contrasts  $C1 (1\ 0)$  and  $C2 (0\ 1)$ . We may be interested in only those voxels which are significantly "active" for both contrasts

# Contrast Masking



- Rather than masking with voxels which survive thresholding, it may be desirable to mask using positive z statistic voxels instead



For example, say that we have two t contrasts C3 (1 -1) and C1 (1 0). It may be desirable to see those voxels for which EV1 is bigger than EV2, only when EV1 is positive

# Factorial design

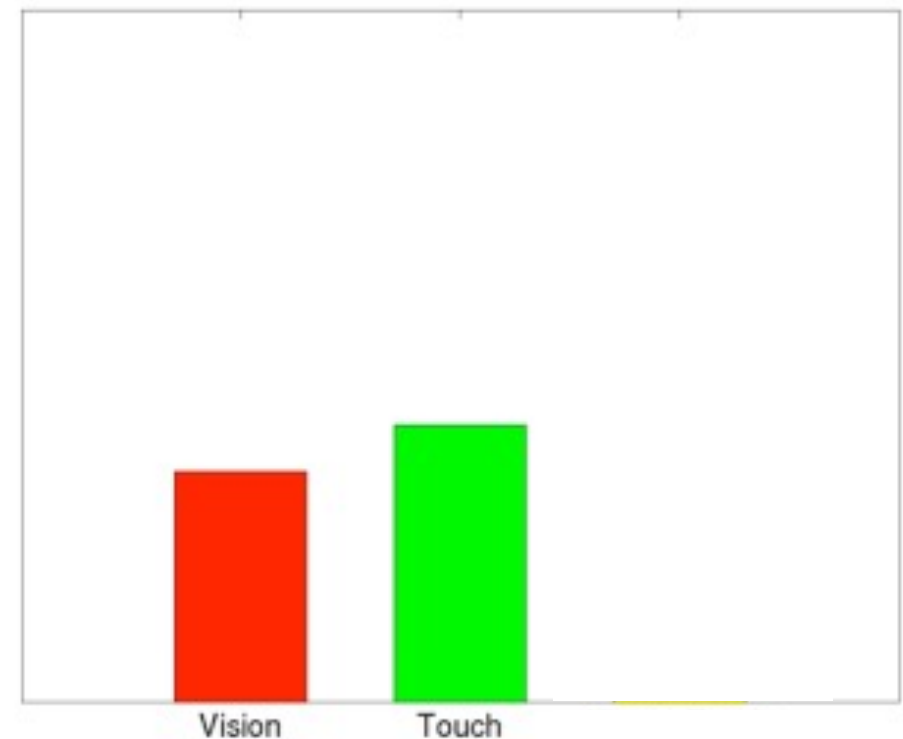
	No Vision	Vision
No Touch		
Touch		

- Allows you to characterise interactions between component processes
- i.e. effect that one component has on another

# No Interaction Effect



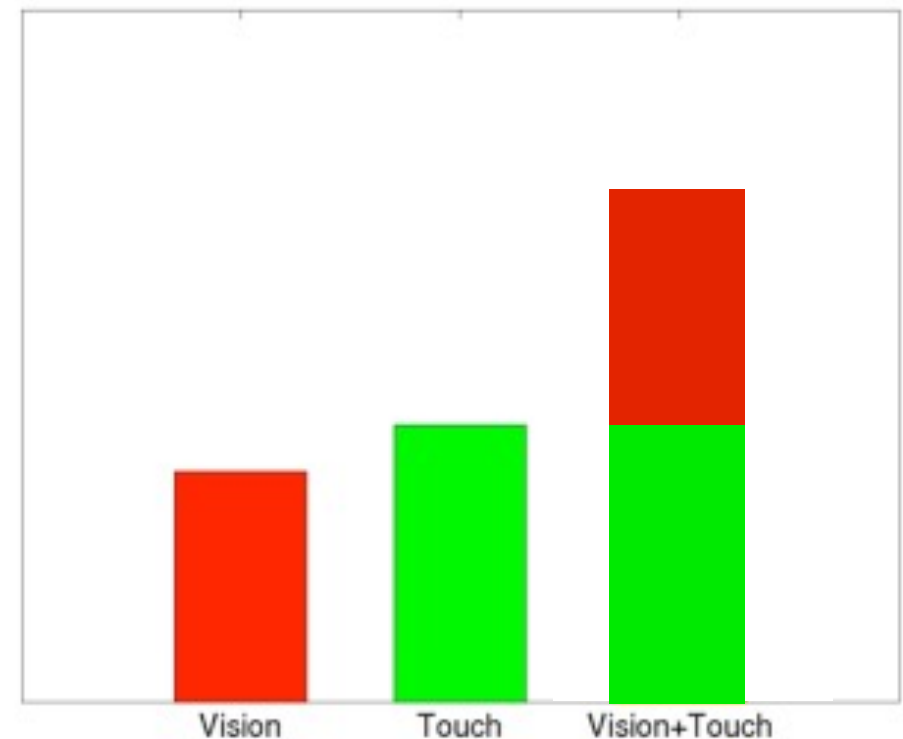
	No Vision	Vision
No Touch		
Touch		



# No Interaction Effect



	No Vision	Vision
No Touch		
Touch		

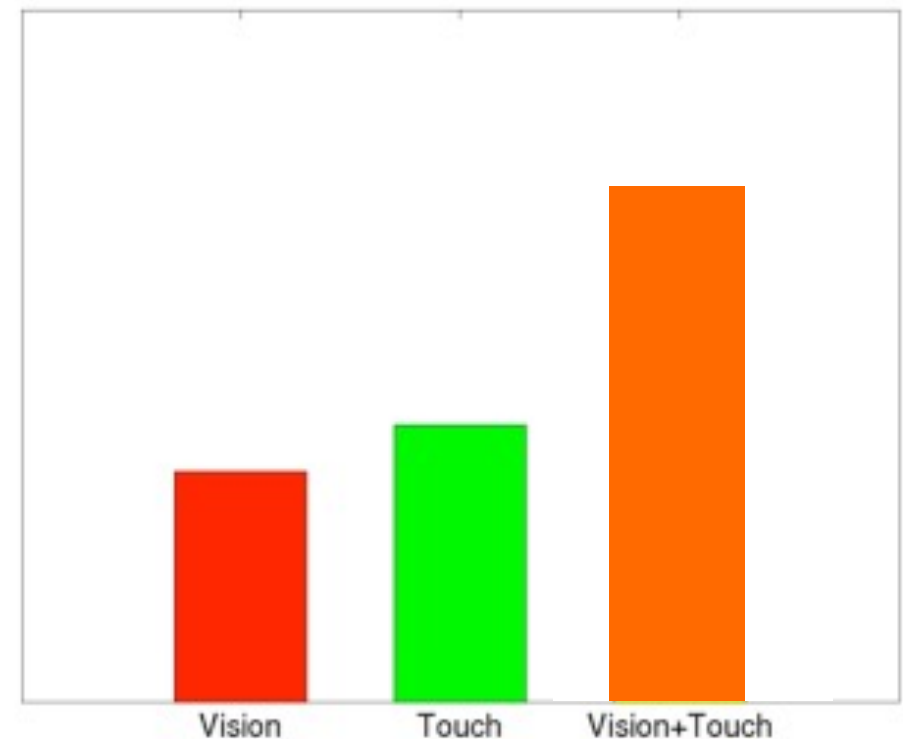




# No Interaction Effect



	No Vision	Vision
No Touch		
Touch		

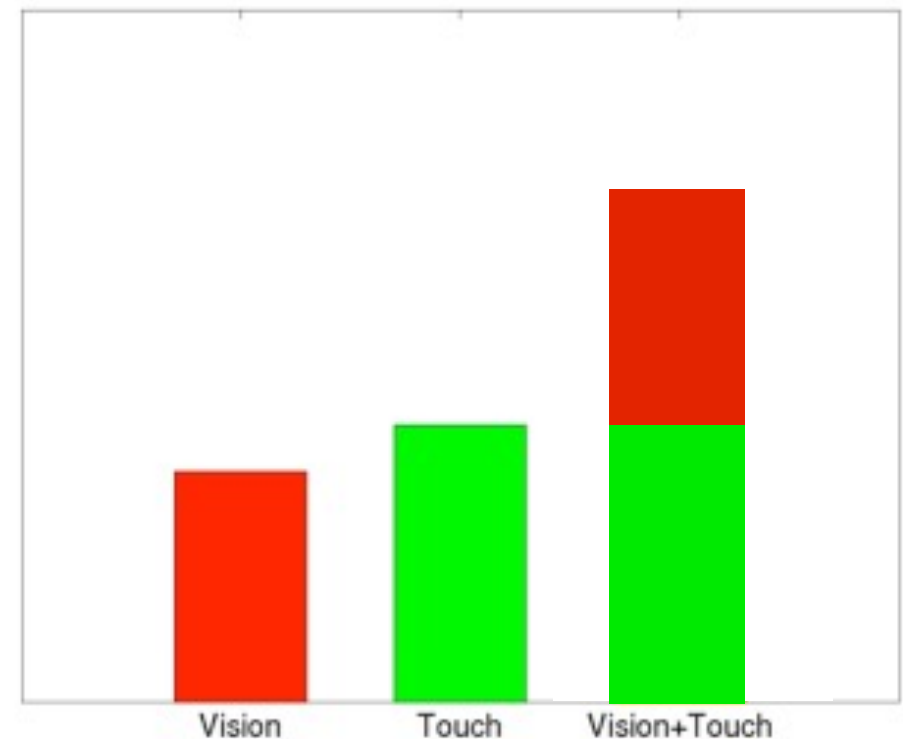


No interaction -  
effects add linearly

# Positive Interaction Effect



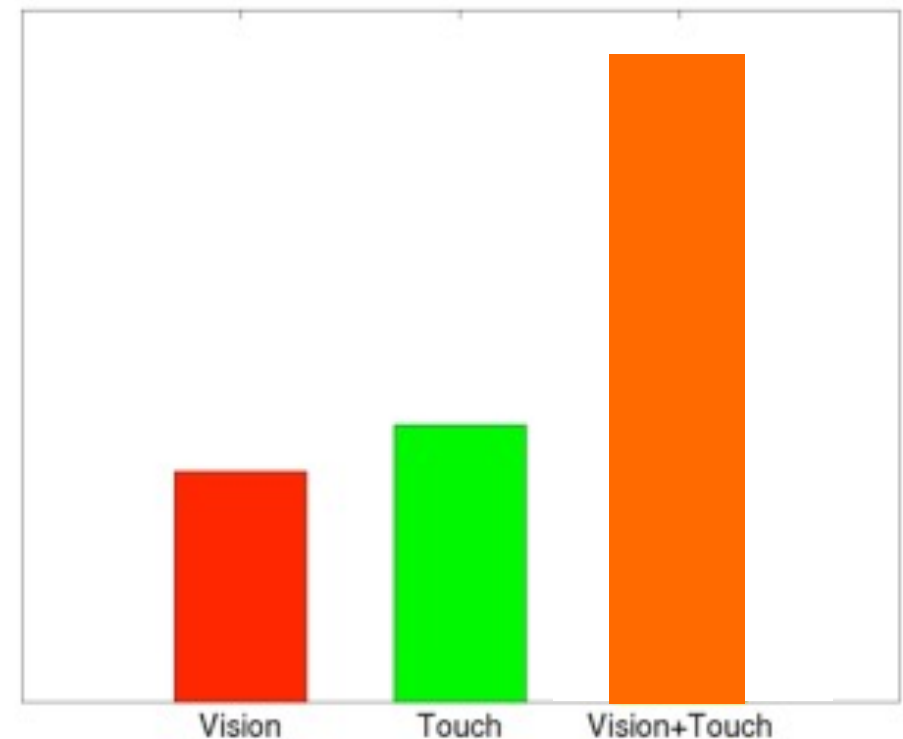
	No Vision	Vision
No Touch		
Touch		



# Positive Interaction Effect



	No Vision	Vision
No Touch		
Touch		

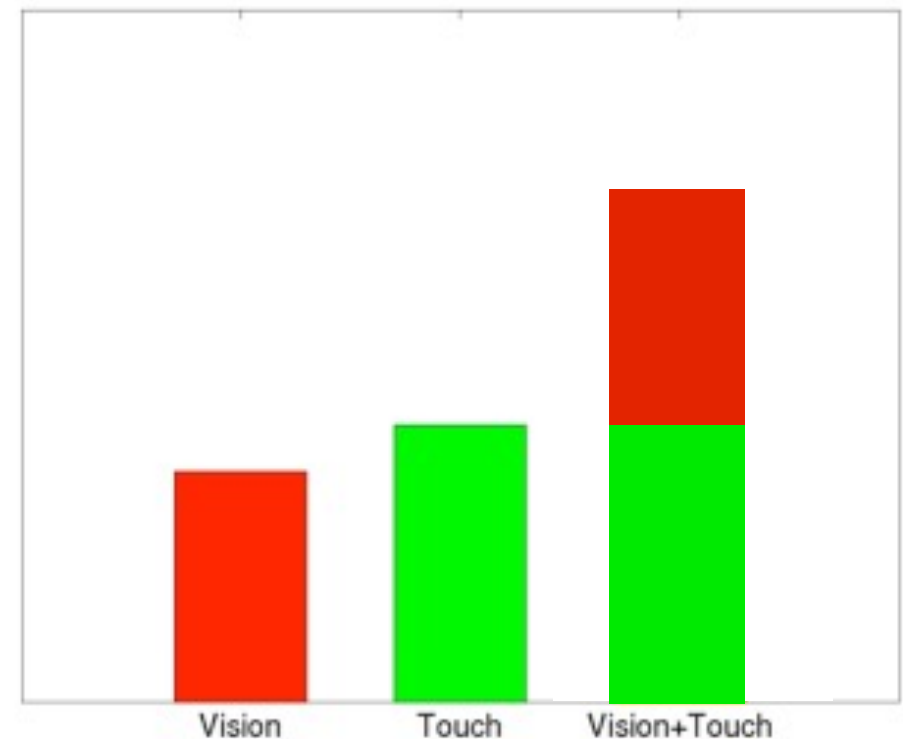


Positive interaction -  
“superadditive”

# Negative Interaction Effect



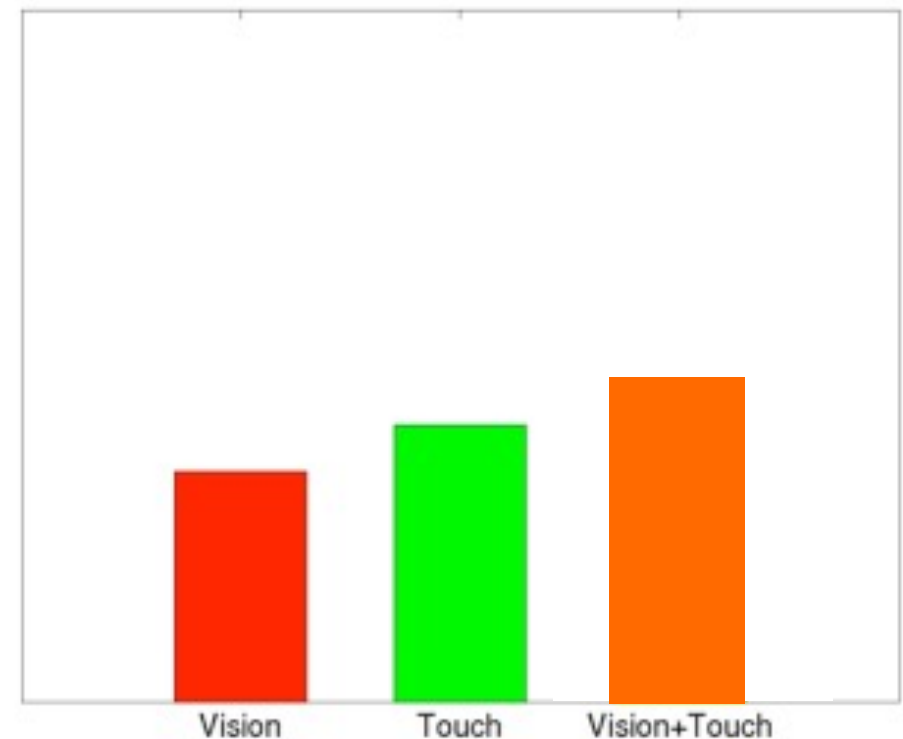
	No Vision	Vision
No Touch		
Touch		



# Negative Interaction Effect



	No Vision	Vision
No Touch		
Touch		

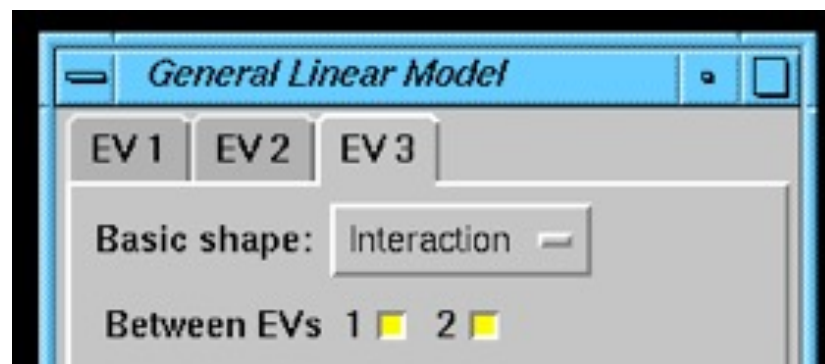


Negative interaction  
- “subadditive”

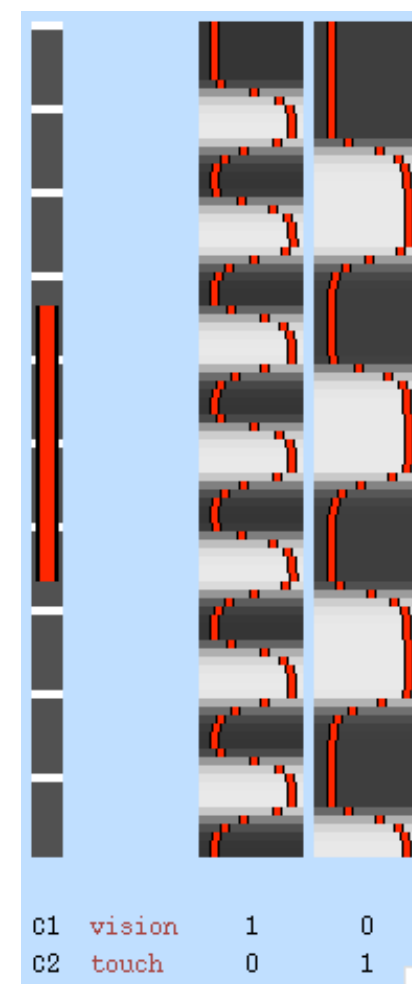
# Modelling Interactions Between EVs



	No Vision	Vision
No Touch		
Touch		



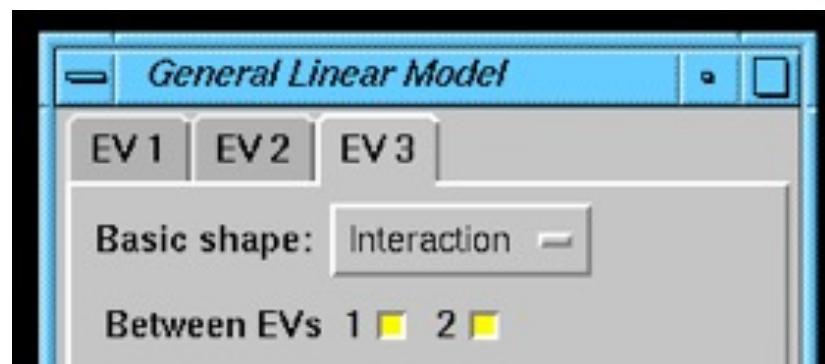
- EV1 models vision on/off
- EV2 models touch on/off



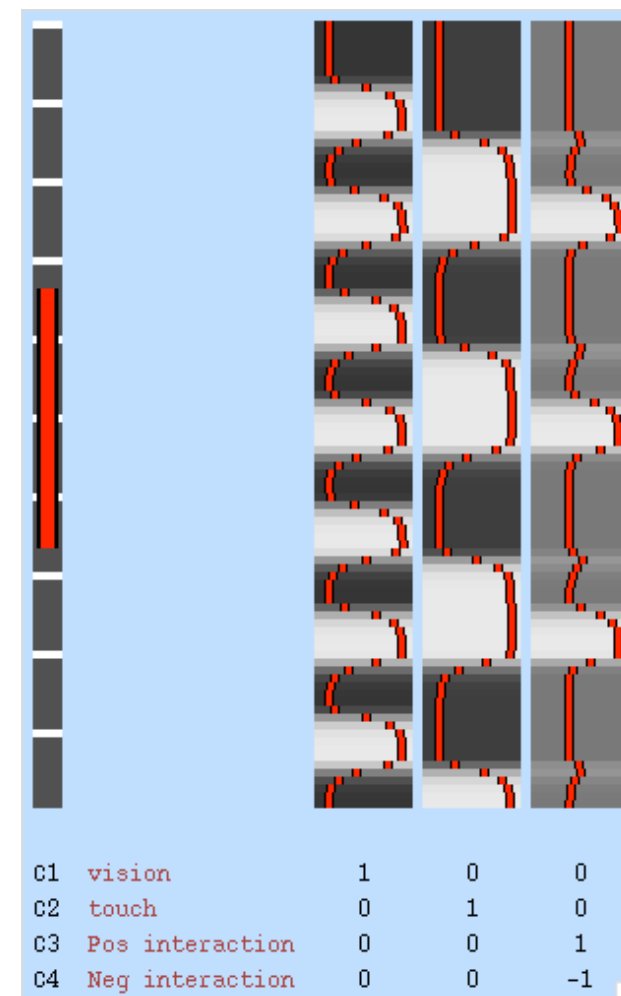
# Modelling Interactions Between EVs



	No Vision	Vision
No Touch		
Touch		



- EV1 models vision on/off
- EV2 models touch on/off
- EV3 Models interaction





# Advanced Analysis: Factorial Designs and Interactions

## Summary:

- Contrast masking allows questions of the form “*A and B*” to be asked
  - F-tests ask “*A or B or both*”
- Factorial design covers different combinations including the interaction
- Interaction can be positive, negative or none and is tested using an extra EV and a simple contrast





# Advanced Analysis: Correlation of EVs and Design Efficiency

# Correlation of EVs

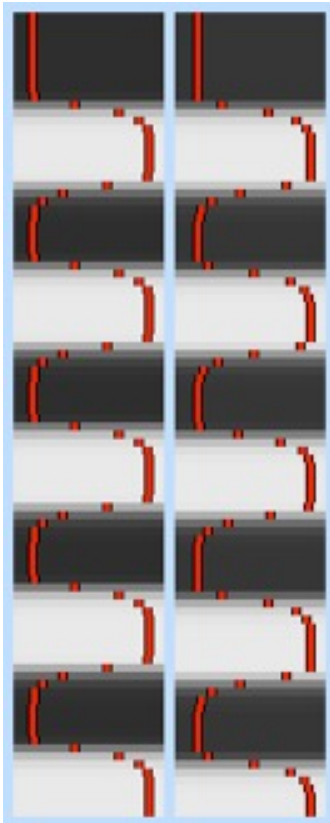


- Correlated EVs are relatively common, but **strong correlation is a problem** in either first-level or group-level designs.
- When EVs are correlated, it is the **unique contribution** from each EV that determines the model's fit to the data and the statistics.
- Start by looking at first-level examples:
  - correlation and rank deficiency
  - design efficiency tool

# Design Matrix Rank Deficiency



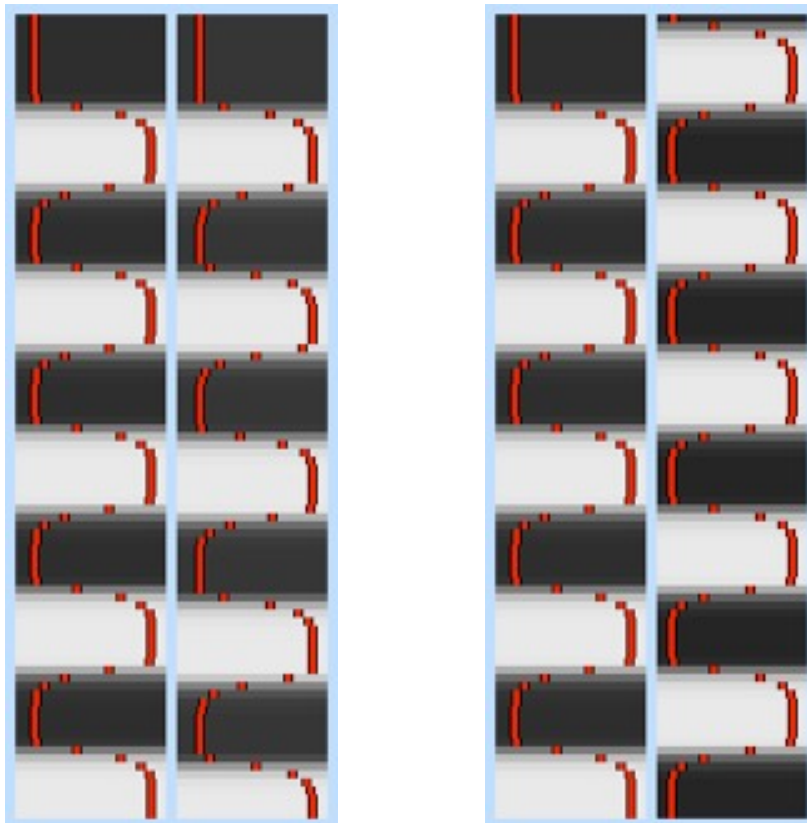
- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. visual and tactile stimulation occurs at very similar times, so it is not possible to separate the responses!



# Design Matrix Rank Deficiency



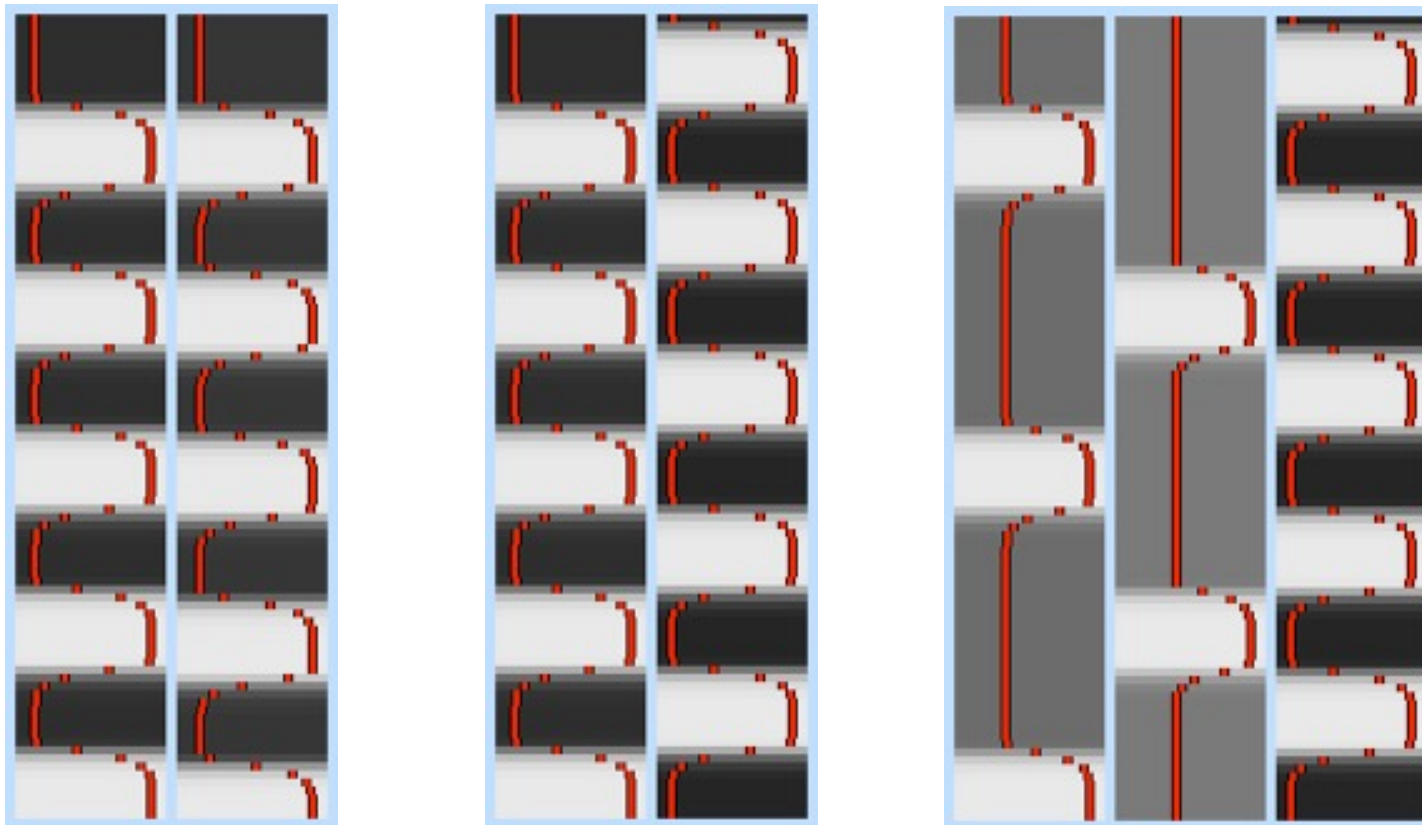
- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. visual and tactile stimulations are exactly opposed (so no baseline)



# Design Matrix Rank Deficiency



- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. modelling visual, tactile, and rest (the last one is effectively baseline and shouldn't be modelled in FSL)

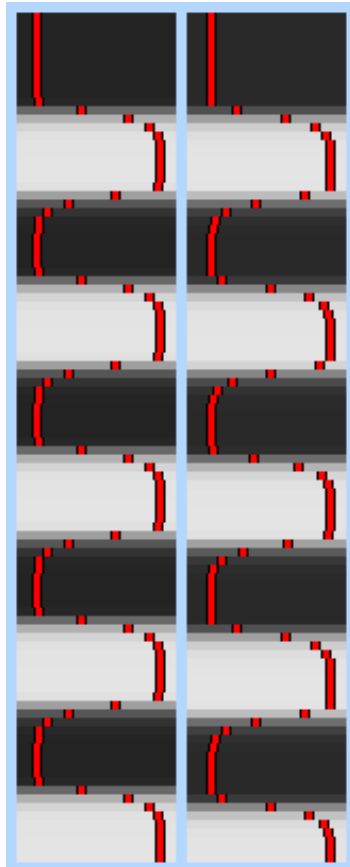


# Close to Rank Deficient Design Matrices



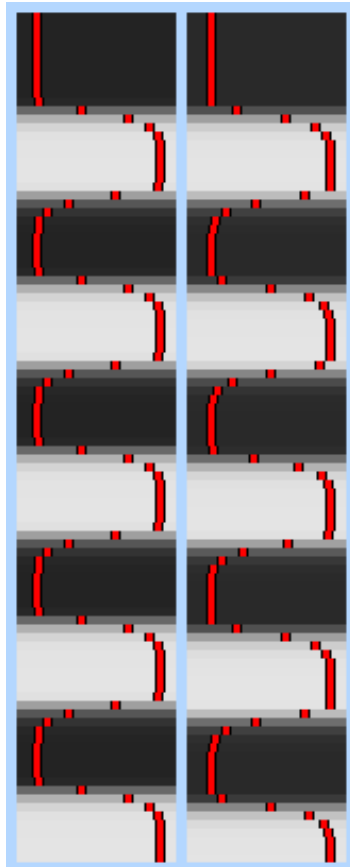
- **Good News:** The statistics always take care of being close to rank deficient
- **Bad News:** the ignorant experimenter may have found no significant effect, because:
  - a) Effect size was too small.
  - b) Being close to rank deficient meant finding an effect would have required a HUGE effect size  
e.g. may need a lot of data to determine how two EVs with very similar timings best combine to explain the data.

# When do we have a problem?



- Depends on SNR, and **crucially** the contrasts we are interested in:
- $[1 \ -1]$  e.g. vis-tact??
- $[1 \ 1]$  e.g. average response??
- $[1 \ 0]$  or  $[0 \ 1]$  ?? e.g. visual? or tactile?

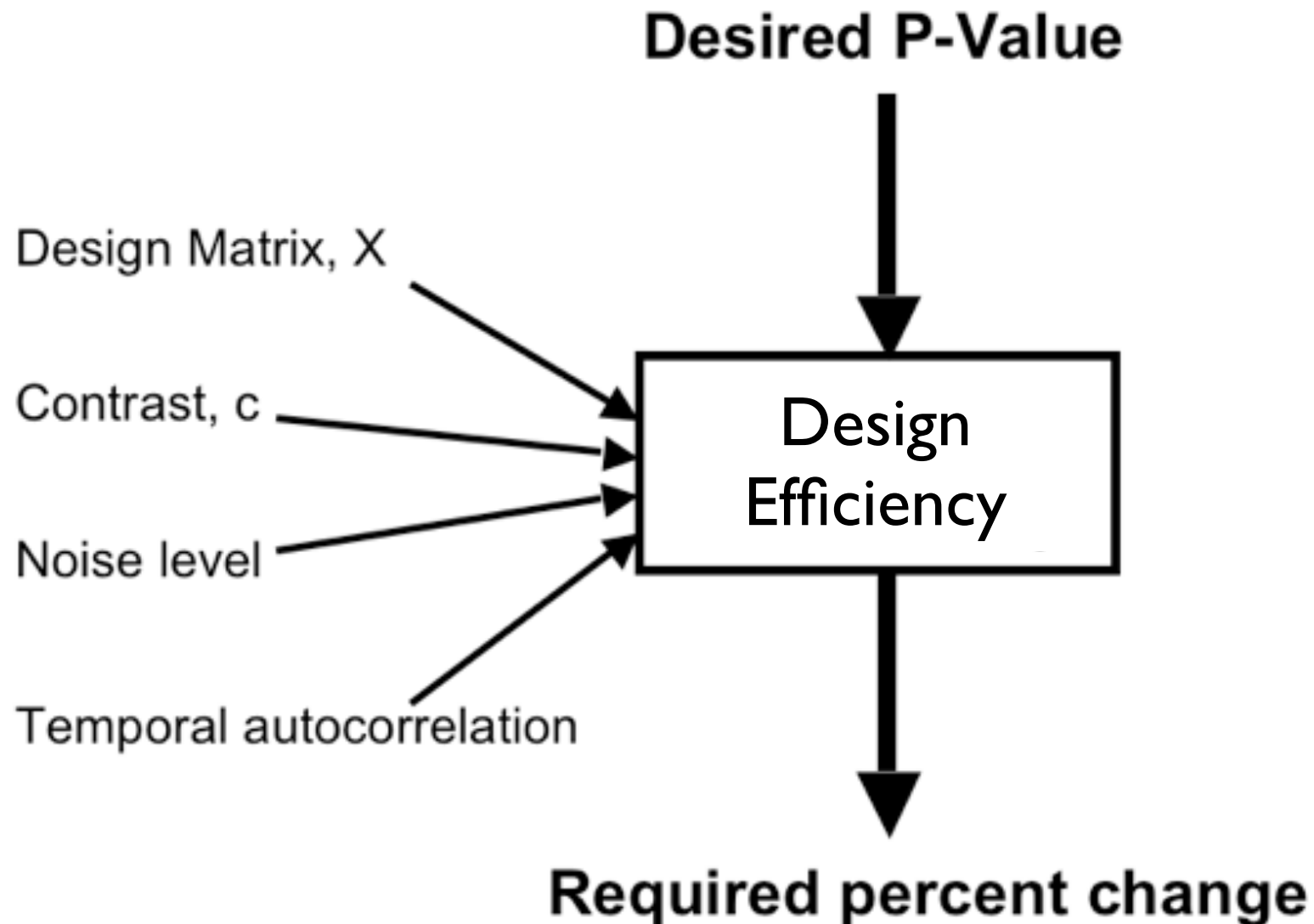
# When do we have a problem?



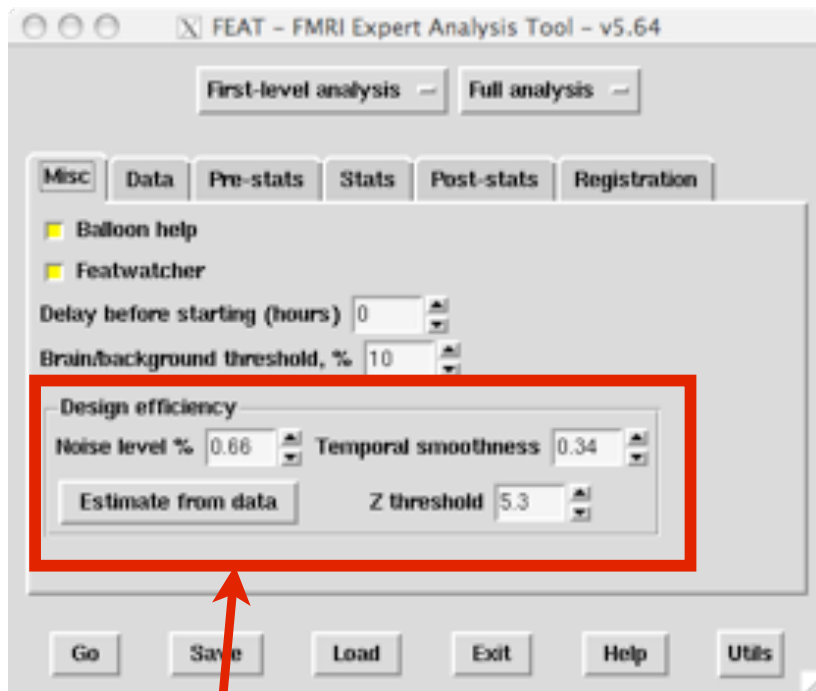
- Depends on SNR, and **crucially** the contrasts we are interested in:
- $[1 \ -1]$  e.g. vis-tact??
  - no chance
- $[1 \ 1]$  e.g. average response??
  - no problem
- $[1 \ 0]$  or  $[0 \ 1]$  ?? e.g. visual? or tactile?
  - no chance



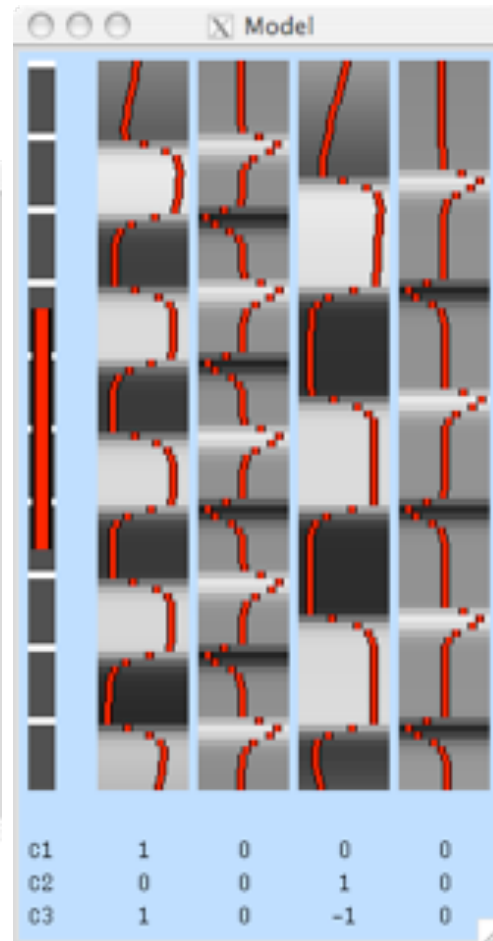
# Design Efficiency



# Design Efficiency



Settings for design efficiency calculations



These are the most useful!

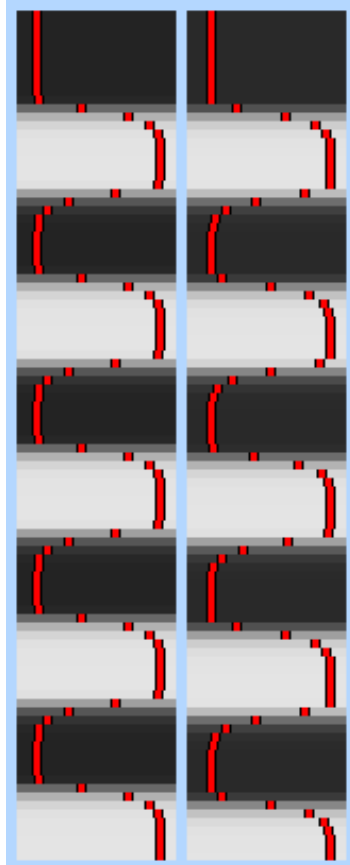
Correlation matrix

Eigenvalues



% change required for each contrast to pass specified z-threshold

# When do we have a problem?



- Depends on SNR, and **crucially** the contrasts we are interested in:
- $[1 \ -1]$  e.g. vis-tact?? Effect size required  
- no chance: **5.3%**
- $[1 \ 1]$  e.g. average response??  
- no problems: **0.84%**
- $[1 \ 0]$  or  $[0 \ 1]$  ?? e.g. visual? or tactile?  
- no chance: **5.3%**

# Case Study: Correlated EVs



## Scenario:

Investigating whether there is a relationship between a patient's disease/behavioural scores and their BOLD responses

## Problem:

Different scores are likely to be strongly correlated.

Which regions' responses correlate with disease scores but not age?

## Solutions:

Combination of F-tests and t-tests

# Correlations, Covariates & Corrections



- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
  - ▶ where we want to 'correct' for age

# Correlations, Covariates & Corrections



- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
    - ▶ where we want to 'correct' for age
  - ▶ If there is correlation between DD and age then it becomes tricky
  - ▶ One option is orthogonalisation of DD and age ...

# Orthogonalisation



# Orthogonalisation



**DON'T DO IT!**



# A better alternative to orthogonalisation



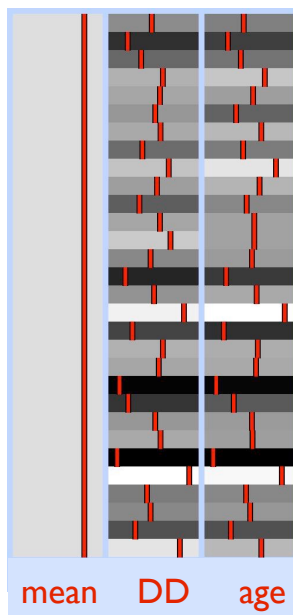
- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
  - ▶ where we want to 'correct' for age

# A better alternative to orthogonalisation



- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
  - ▶ where we want to 'correct' for age

t-test



[ 0 | 0 ]

A t-test for a single EV is determined only by variability in BOLD signal that *cannot* be accounted for by other EVs.

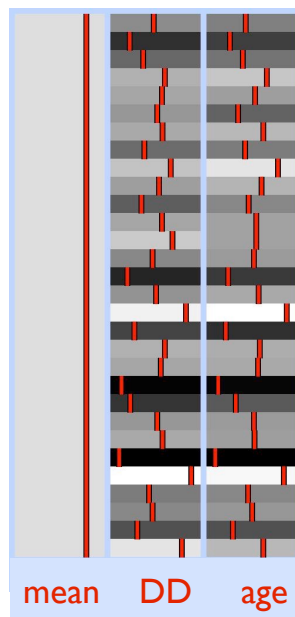
This is a **conservative** result: only when DD can *uniquely* explain the measurements will there be a significant result.

# A better alternative to orthogonalisation



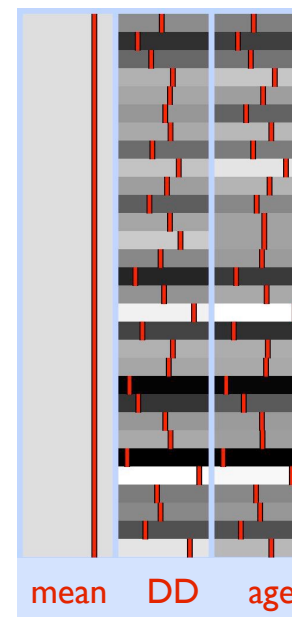
- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
  - ▶ where we want to 'correct' for age

t-test



$$[0 \quad 1 \quad 0]$$

F-test



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

□  
■  
■

# A better alternative to orthogonalisation

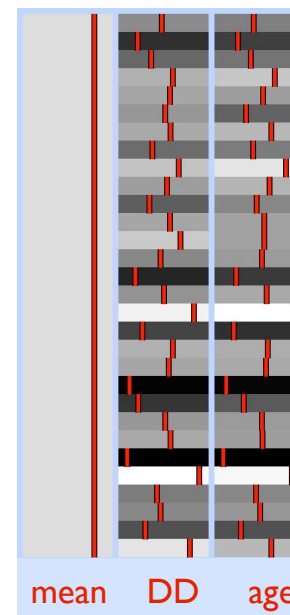


- Consider a case example:
  - ▶ Disease Duration (DD) + age (demeaned)
  - ▶ where we want to 'correct' for age

## F-test

An F-test finds regions where signal can be explained by *any combination* of EVs.

Will show significant results where *either DD or age or both* can explain the measurements.

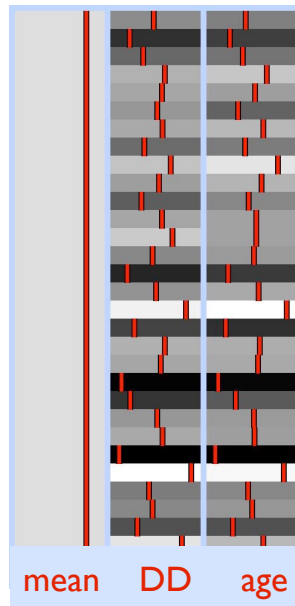


$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad \square$$
$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad \blacksquare$$
$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad \blacksquare$$

# A better alternative to orthogonalisation

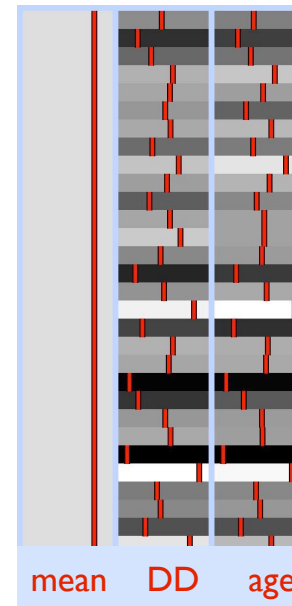


t-test



$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

F-test



$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad \square$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad \blacksquare$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad \blacksquare$$

Results (a fairly typical example with strong correlation):

Not significant (t-test)

Significant (F-test)

Interpretation: Significant correlation with *both* DD and age, but cannot separate the effects as they are too highly correlated and the response to unique portions (if any) are too weak.

Follow on: one way to (potentially) separate the effects would be to recruit new subjects such that DD and age were less correlated (need more data to go beyond the above interpretation).



# Advanced Analysis: Correlated EVs

## Summary:

- Correlation of EVs makes it difficult for the GLM to assign unique contributions and often leads to no significant results
- Extreme correlation gives rank deficiency
- Problem of correlation depends on the contrast
- Design efficiency gives required % BOLD change to get a significant result *per contrast* (like power calc.)
- Can also get info about where correlations are
- Orthogonalisation: DON'T DO IT!
- In practice consider F-tests for combined explanatory results as well a t-test (unique contributions)
- Try to break correlations through planning/recruitment



# That's All Folks